

Centralized, Distributed, and Coalitional Model Predictive Control

José M. Maestre



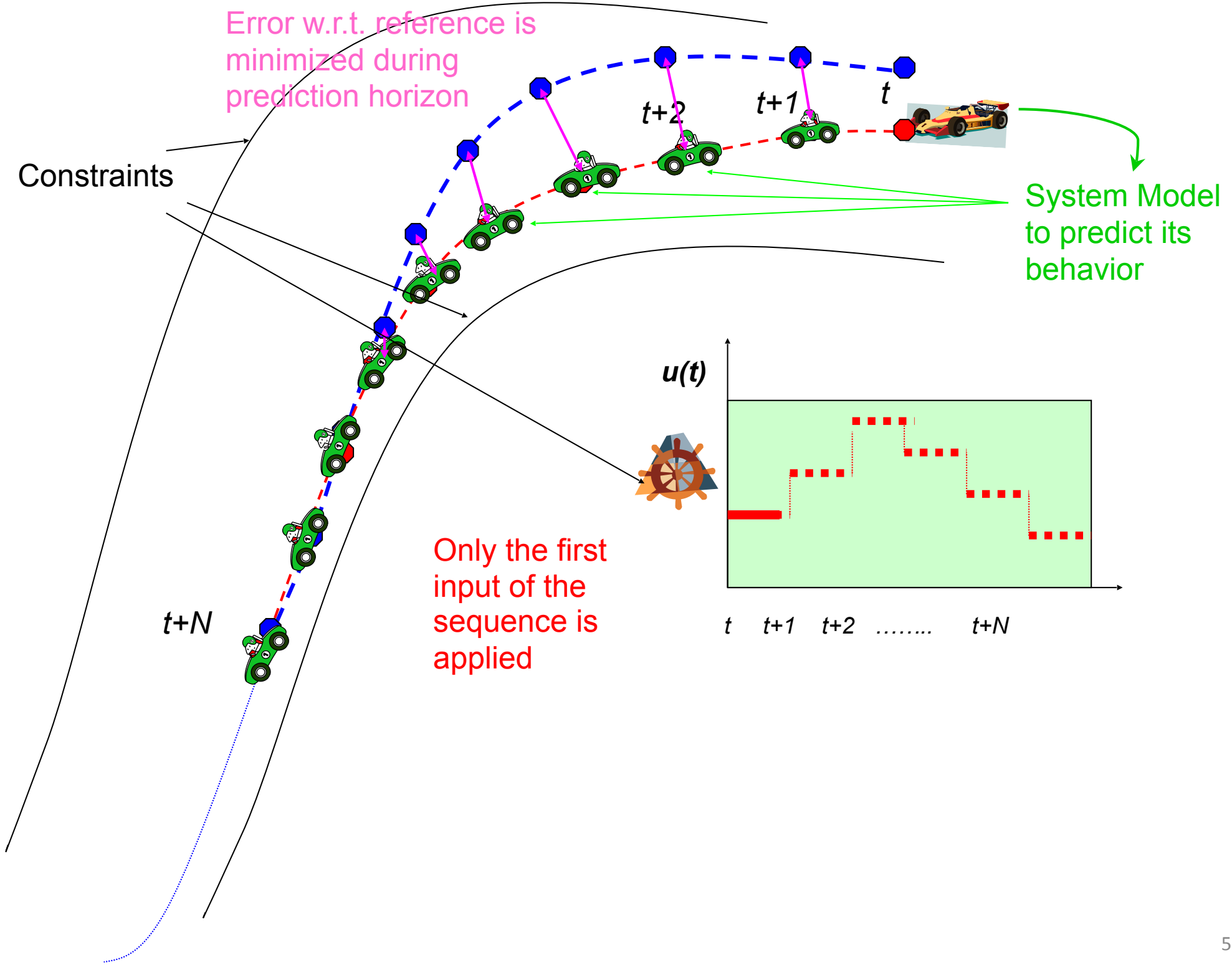


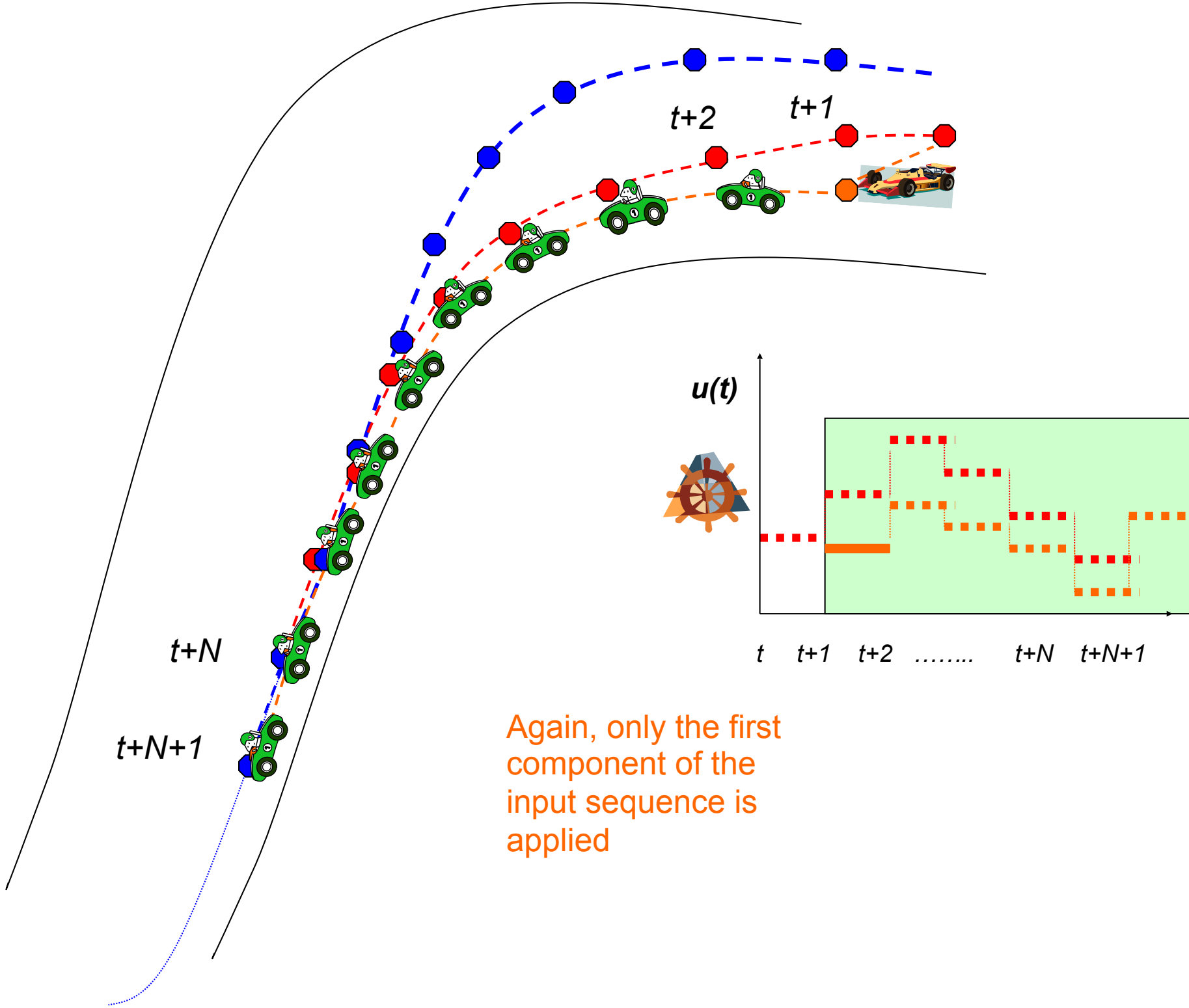
Outline

- Model Predictive Control
- Distributed Model Predictive Control
- Coalitional Model Predictive Control
- Conclusions

Outline

- **Model Predictive Control**
- Distributed Model Predictive Control
- Coalitional Model Predictive Control
- Conclusions





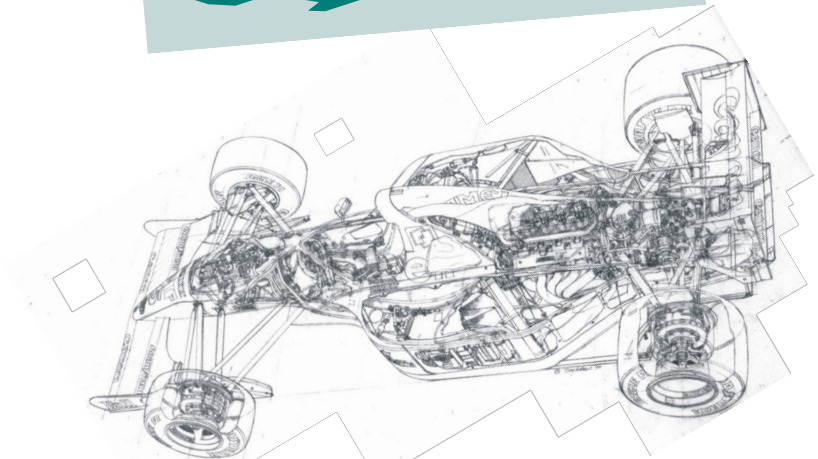
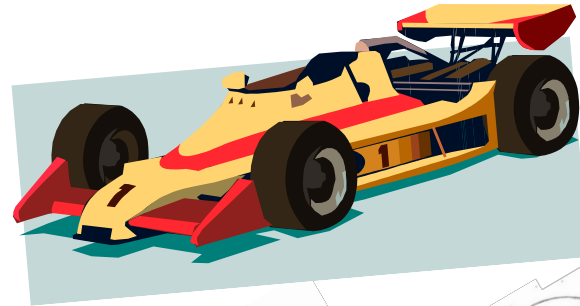
Model Predictive Control

- What is it needed to apply MPC?
 - System model available
 - A cost function that involves the controlled and manipulated variables
 - Prediction horizon (tuning parameter)
 - Additional information to take into account: disturbances, constraints, delays



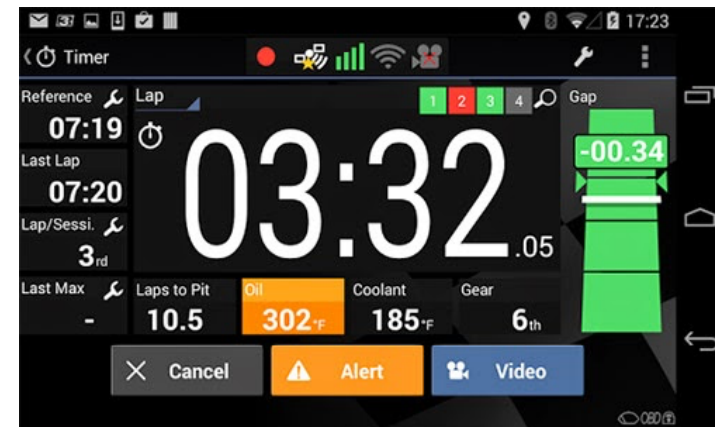
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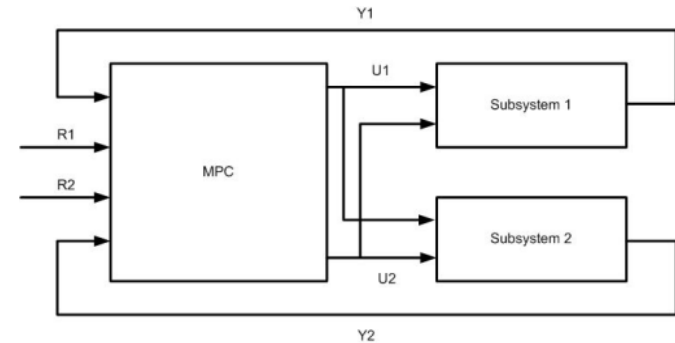
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$$\{U_1^c(t), U_2^c(t)\} = \arg \min_{U_1, U_2} J_1(x_1(t), U_1, U_2) + J_2(x_1(t), U_2, U_1)$$

$$x_{1,k+1} = A_1 x_{1,k} + B_{11} u_{1,k} + B_{12} u_{2,k}$$

$$x_{1,0} = x_1(t)$$

$$x_{1,k} \in \mathcal{X}_1, k = 0, \dots, N$$

$$u_{1,k} \in \mathcal{U}_1, k = 0, \dots, N-1$$

$$x_{2,k+1} = A_2 x_{2,k} + B_{22} u_{2,k} + B_{21} u_{1,k}$$

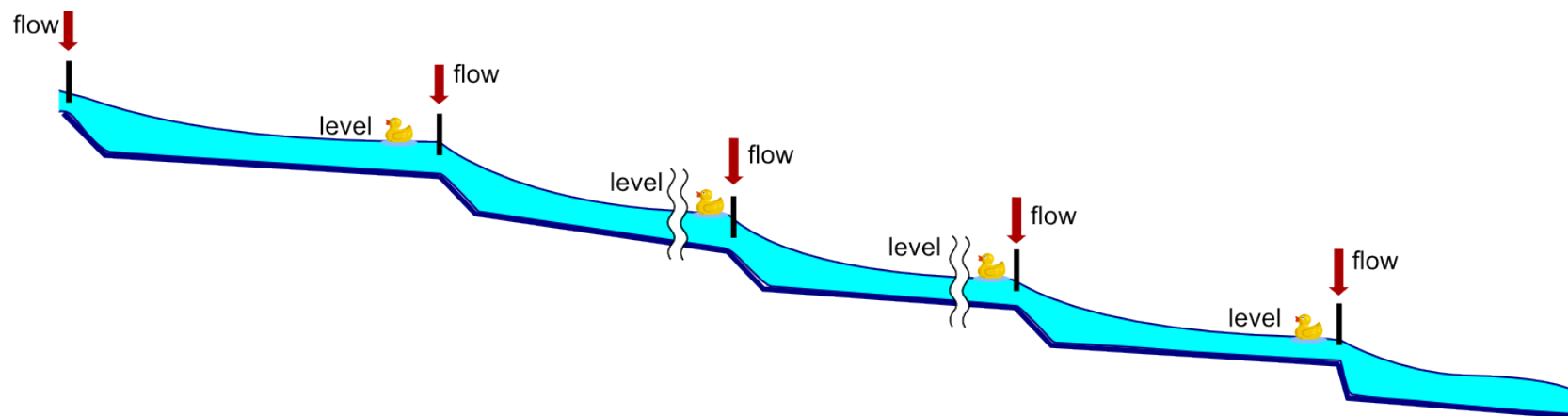
$$x_{2,0} = x_2(t)$$

$$x_{2,k} \in \mathcal{X}_2, k = 0, \dots, N$$

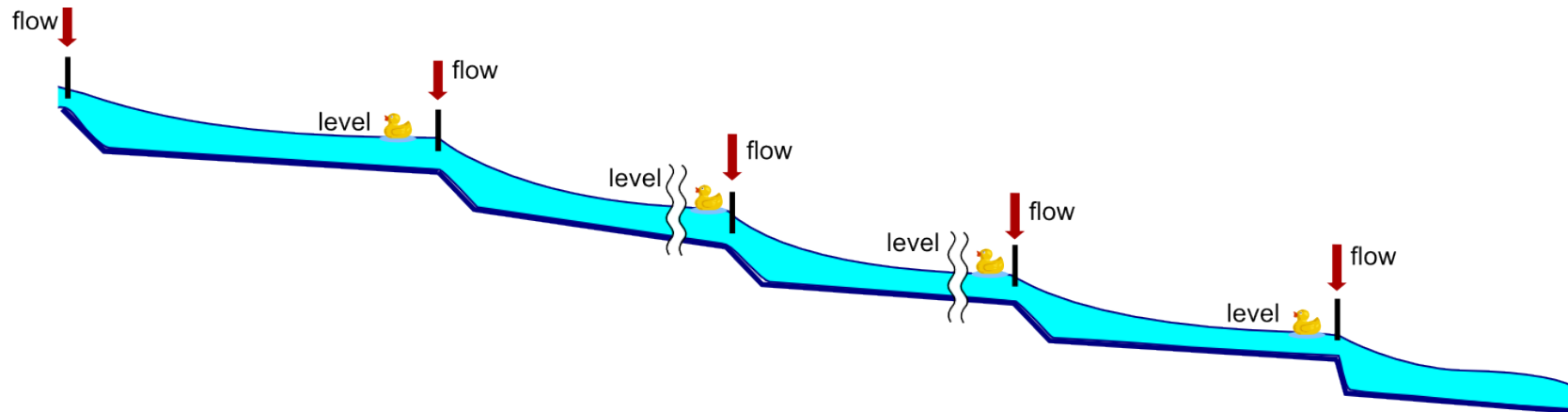
$$u_{2,k} \in \mathcal{U}_2, k = 0, \dots, N-1$$

Example (I)

- Irrigation canals are waterways used to deliver water to farmers
- 40% of all the food production comes from irrigated lands and 70% of freshwater consumption is used in agriculture (F.A.O.)



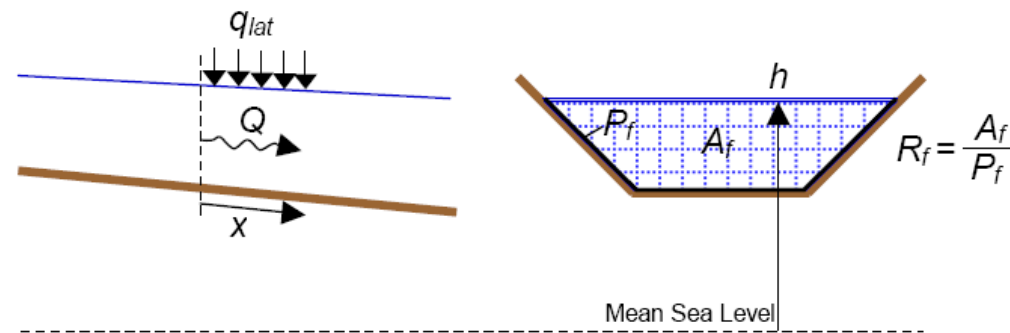
Example (I)



- The control objectives in this context are generally:
 - 1) keeping the water levels at the end of each reach at set point
 - 2) while using as few changes in structure settings as possible or as little energy as possible when pumping water

Example (I)

- The behavior of these systems is well characterized but complex, e.g.:



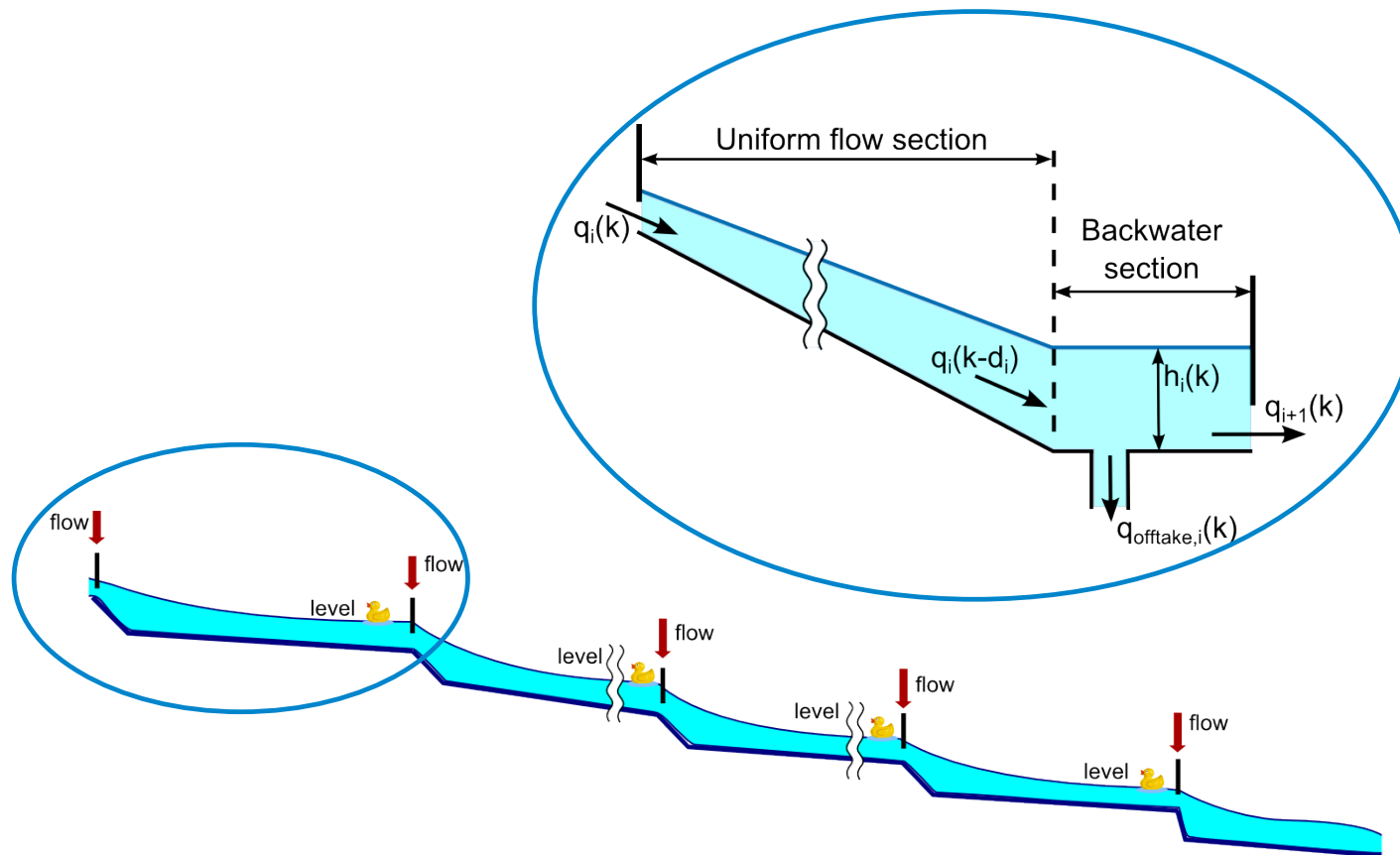
Inertia
Convective acceleration
Gravitational Force
Friction Force

$$\underbrace{\frac{\partial Q}{\partial t}}_{\text{Inertia}} + \underbrace{\frac{\partial}{\partial x} \left(\frac{Q^2}{A_f} \right)}_{\text{Convective acceleration}} + \underbrace{g \cdot A_f \frac{\partial h}{\partial x}}_{\text{Gravitational Force}} - \underbrace{\frac{g \cdot Q |Q|}{C^2 \cdot R_f \cdot A_f}}_{\text{Friction Force}} = 0 \quad \text{Momentum Balance}$$

Partial Differential Saint-Venant Equations

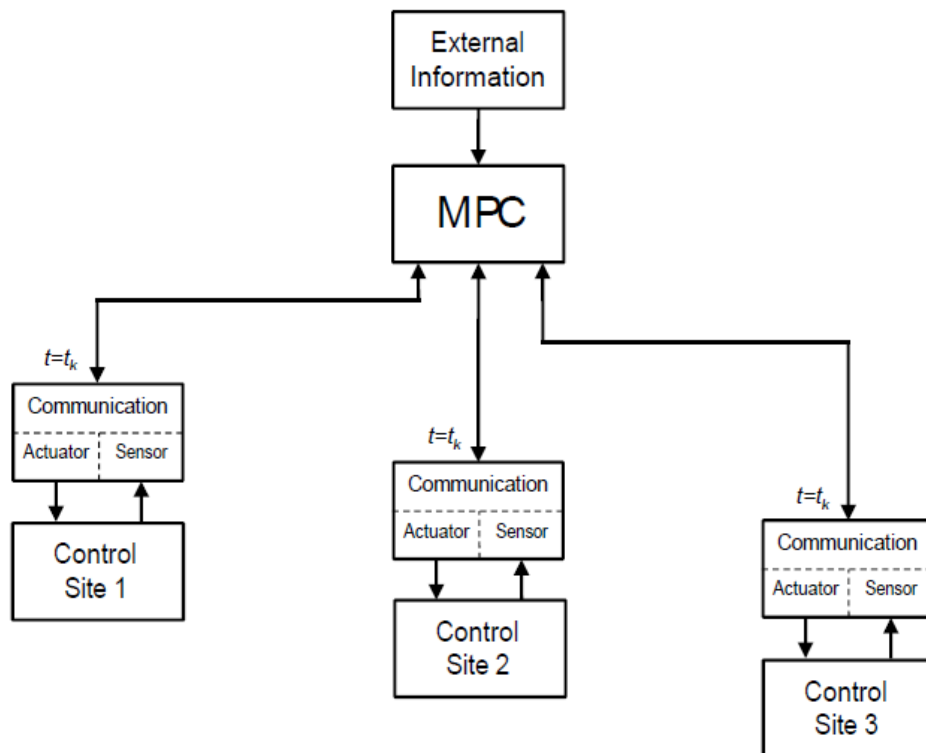
Example (I)

- For this reason, it is common to use simpler models for control



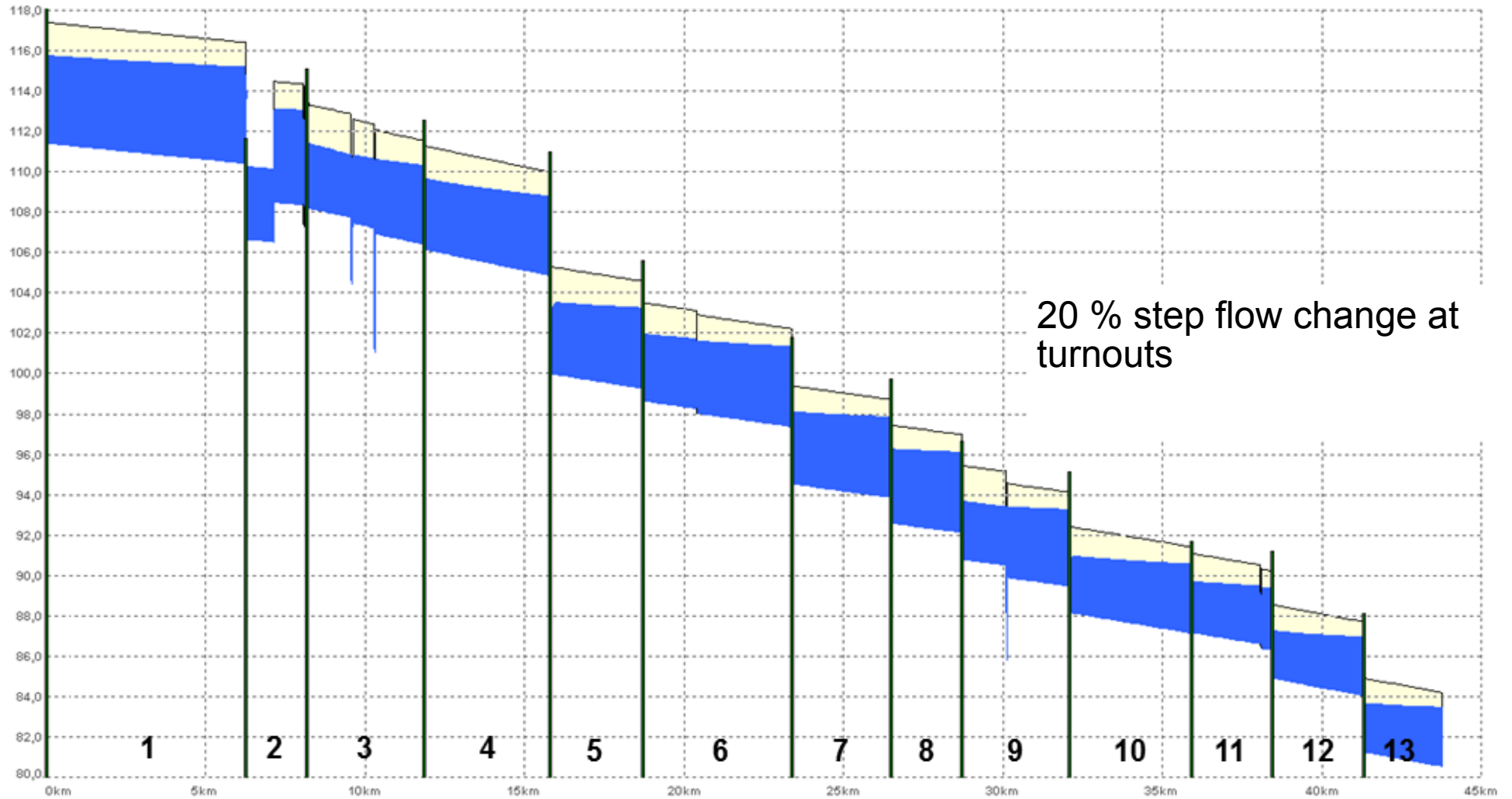
Example (I)

MPC uses the system model to predict its evolution and calculate the optimal inputs along a certain horizon according to a given cost function

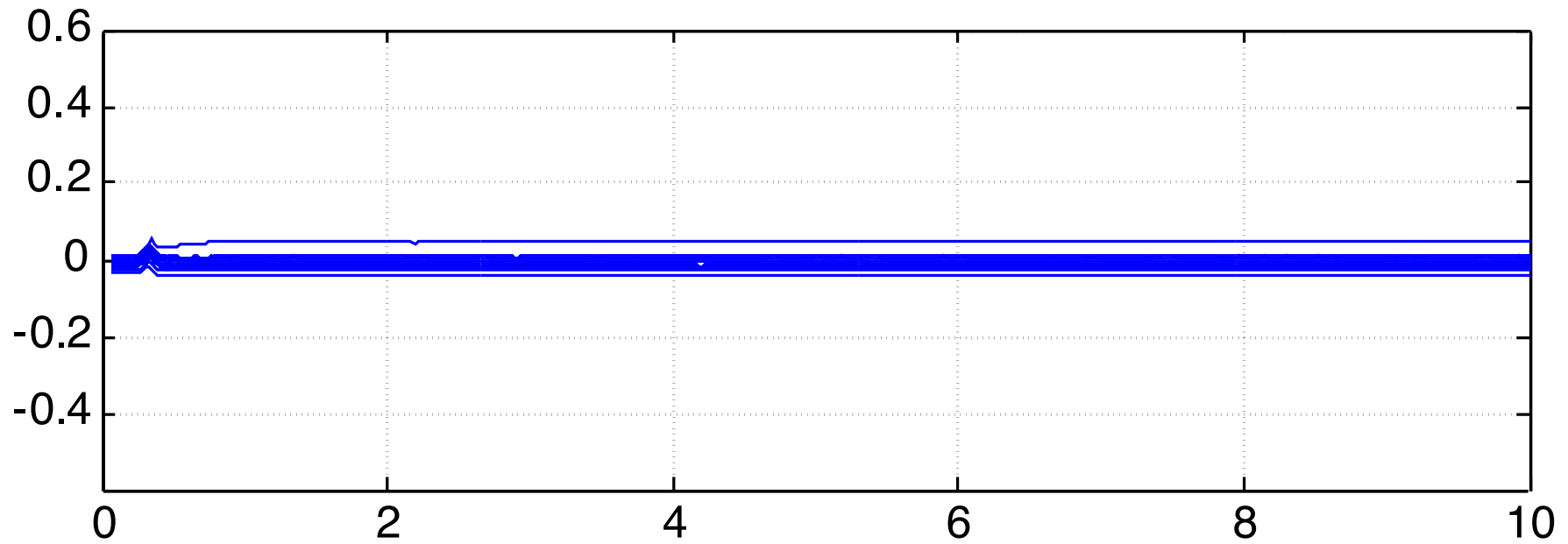


$$u^*(k : k + N_c) = \{u_1^*(k), u_2^*(k), u_3^*(k), u_1^*(k + 1), u_2^*(k + 1), u_3^*(k + 1), \dots, u_1^*(k + N_c), u_2^*(k + N_c), u_3^*(k + N_c)\}$$

Example (I)



Example (I)



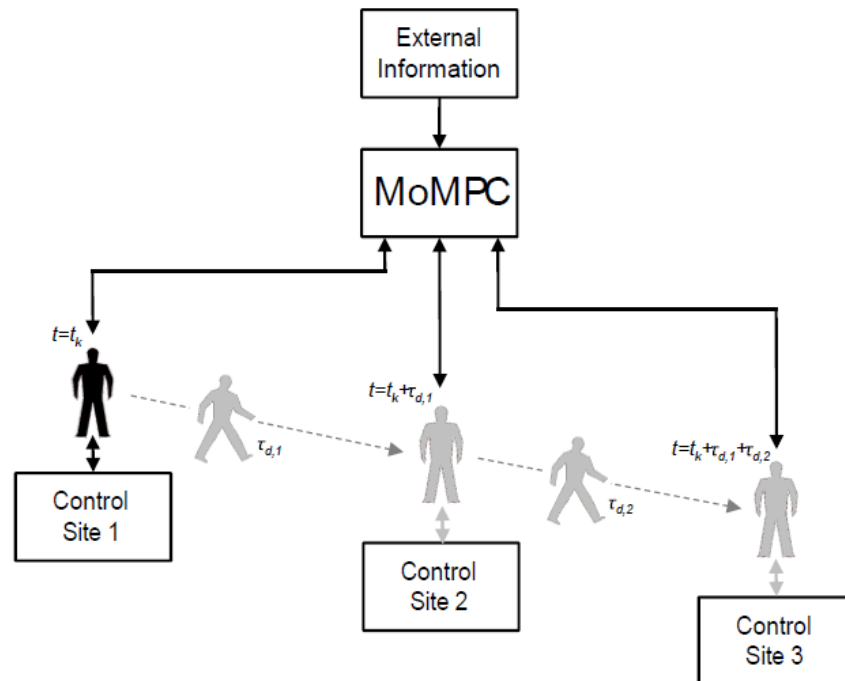
Example (I)

- However, there are certain issues MPC cannot solve...



Example (I)

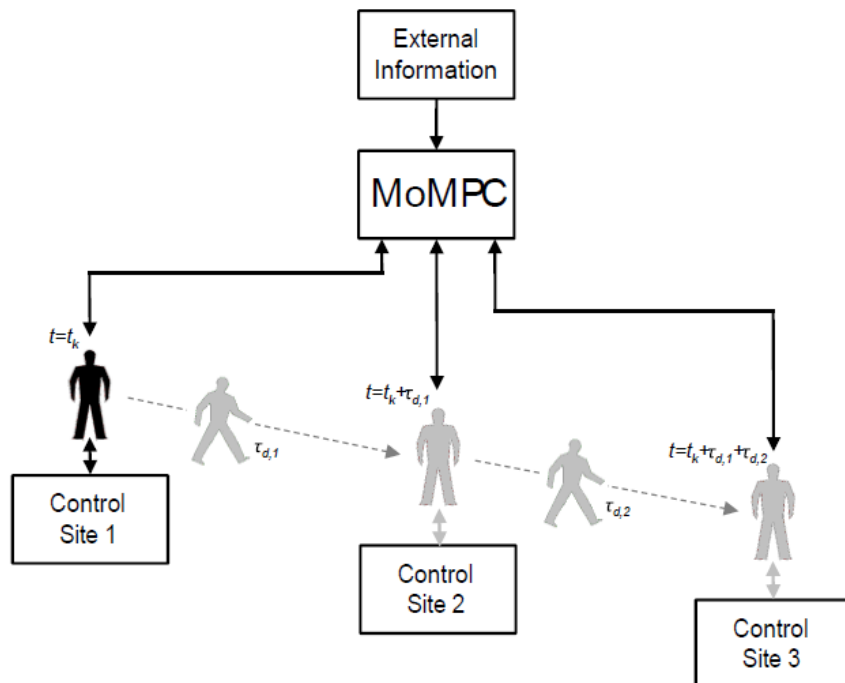
- We integrate the operator inside the MPC controller as a movable sensor/actuator with a delay due to traveling times



$$u^*(k : k + N_c) = \{u_1^*(k), 0, 0, \underbrace{0, 0, 0}_{\text{travel time}}, 0, u_2^*(k + 2), 0, \underbrace{0, 0, 0}_{\text{travel time}}, 0, 0, u_3^*(k + N_c)\}$$

Example (I)

- To this end, an MI-QP is solved in an event triggered fashion:



$$\min_{u(k:k+N_c), p_v^j} \sum_{l=0}^{N_p-1} \ell(k+l)$$

s.t.

$$x(l+1) = Ax(l) + Bu(l) + w(l)$$

$$p_v^j \in \mathcal{P}_v^j(N_s)$$

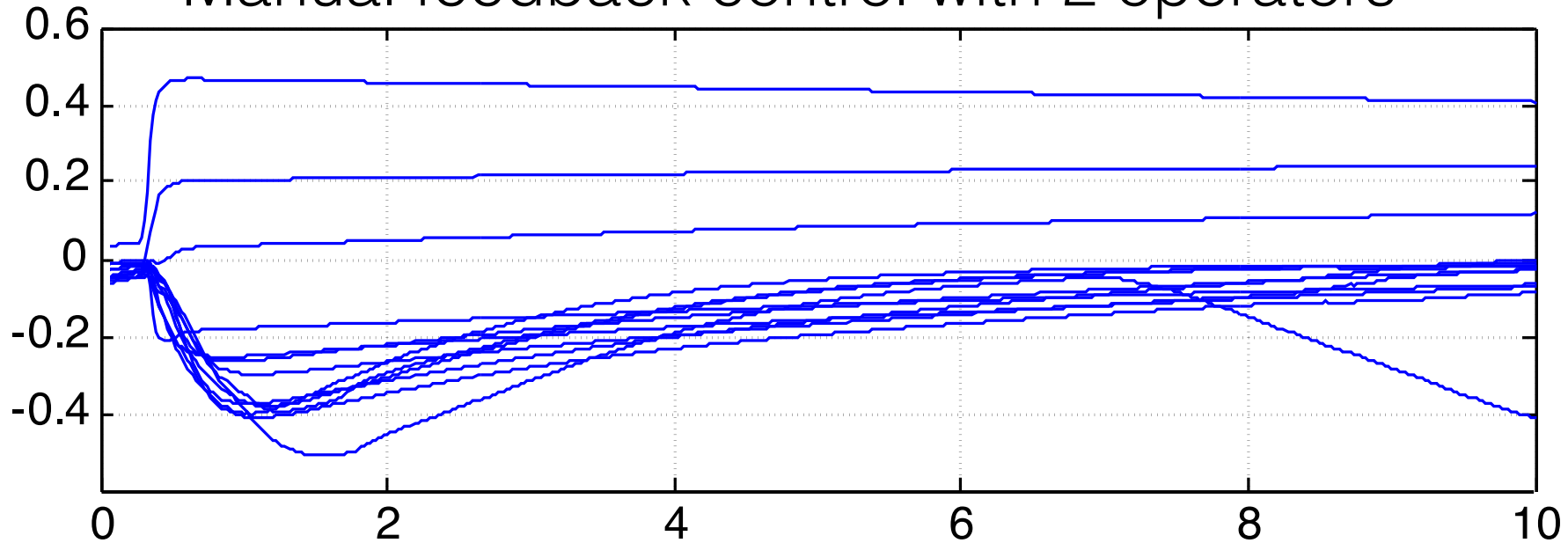
$$u_i(l) = 0, \quad \forall i \in \mathcal{V}, \quad \forall l \in \{k, k+1, \dots, k+N_c\} : a(p_v^j, i, l) = 0$$

$$u_i(l) = 0, \quad \forall i \in \mathcal{V}, \quad \forall l \in \{k, k+1, \dots, k+N_c\} : a(\hat{p}_v^{-j}, i, l) = 0$$

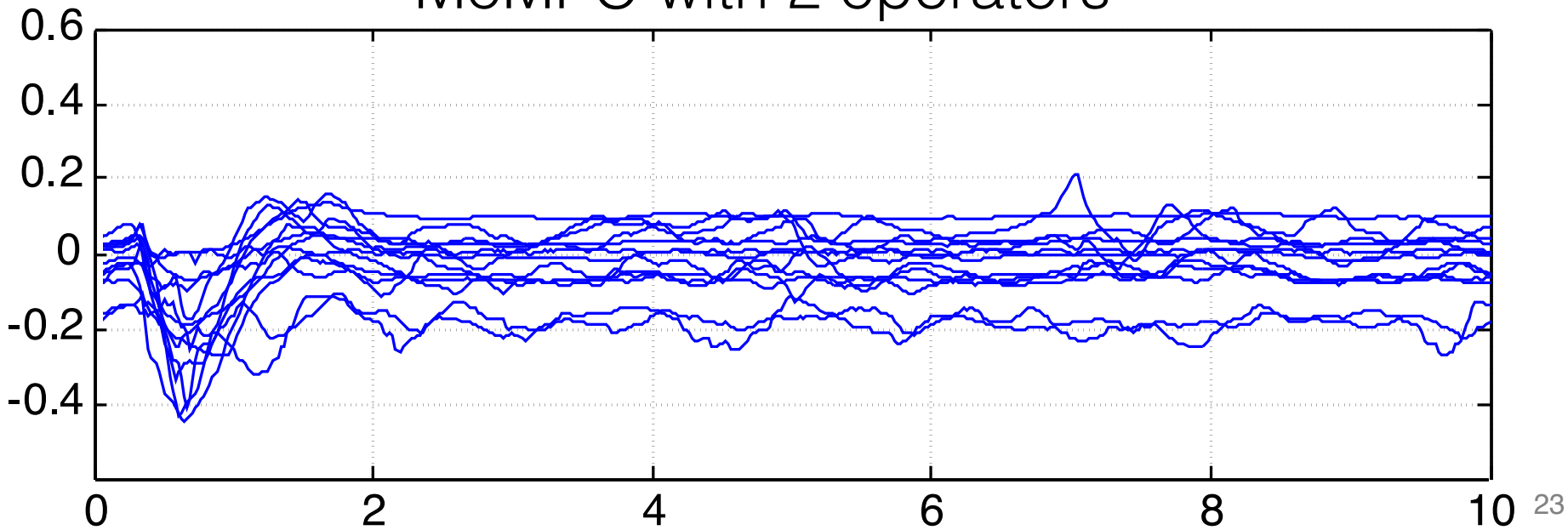
$$x(l) \in \mathcal{X}$$

$$u(l) \in \mathcal{U}$$

Manual feedback control with 2 operators



MoMPC with 2 operators



Example (I)

Water Deficiency Comparison

Control configuration	WD(%)
<i>Centralized Control</i>	1.06
<i>2 Human operators applying feedback control</i>	11.21
<i>Mobile Canal Control with 2 human operators</i>	1.87

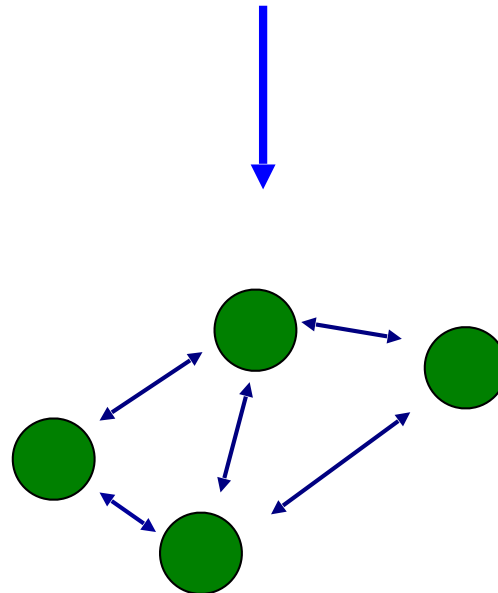
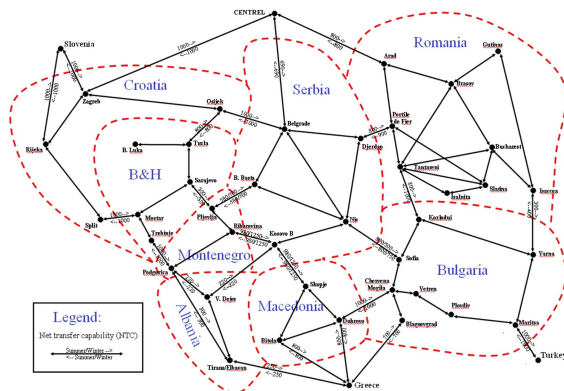
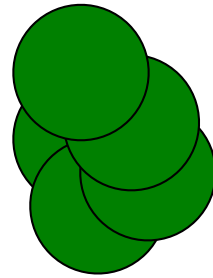
Maestre, J.M.; van Overloop, P.J.; Hashemy, M.; Sadowska, A.; Camacho, E.F., "Human in the loop model Predictive Control: an irrigation canal case study," Decision and Control (CDC), 2014 IEEE 53rd Annual Conference on , vol., no., pp.4881,4886, 15-17 Dec. 2014

P.J. van Overloop, J.M. Maestre, A. Sadowska, E. F. Camacho, B. de Schutter. Human-in-the-Loop Model Predictive Control of an Irrigation Canal. IEEE Control Systems Magazine 07/2015; 35(4):19-29.

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- **Distributed Model Predictive Control**
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- Conclusions

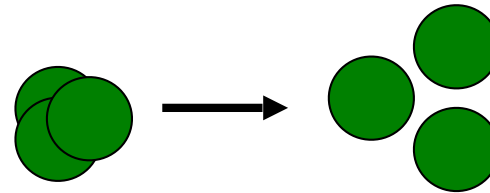
Distributed Model Predictive Control



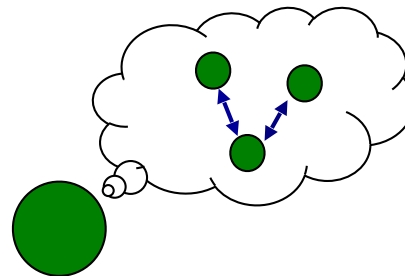
Distributed Model Predictive Control

- Many control schemes have been proposed with differences on

– System decomposition



– Information available



– Communicational constraints

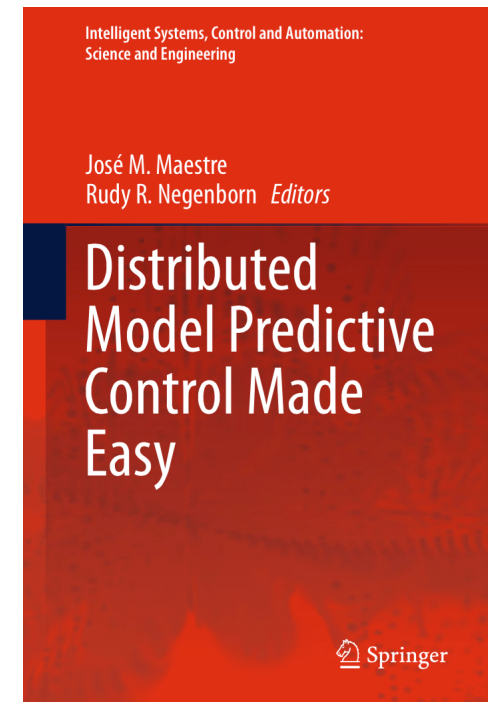
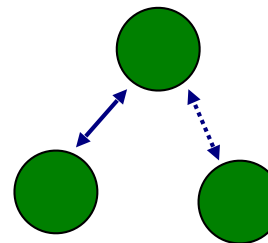


TABLE An overview of distributed model-predictive control (MPC) process commonalities.

		Distributed MPC Commonalities: Process Features		≈ 100%
				≈ 50%
				≈ 0%
System Type	Multiple Autonomous Systems [28]–[47]		Decomposed Monolithical System [48]–[62]	
Process Type	Linear [28], [29], [32], [34], [36]–[38] [40]–[43], [45], [47], [48], [50] [51]–[55], [57]–[62]	Nonlinear [30], [31], [33], [35] [39], [44], [46], [56]	Hybrid [49]	
Type of Model	Transfer Function [60]		State Space [28]–[59], [61], [62]	
Randomness	Deterministic [28], [30]–[32], [34]–[37] [38]–[42], [44]–[47], [49]–[53], [55]–[62]		Nondeterministic [29], [33], [43], [48], [54]	
Type of Control	Regulation [29], [31], [33], [36], [38]–[40] [43]–[45], [48], [50]–[55] [56], [58], [62]	Tracking [28], [30], [32], [34], [41] [42], [47], [49], [53], [59]–[61]	Economic [28], [32], [35], [37] [46], [53], [57]	
Coupling Source	Constraints [28]–[32], [35], [36], [38], [39], [41], [42] [43], [45], [50], [52], [57], [59], [60], [62]		Objective [28], [31], [33], [36], [43], [44], [48], [50] [56], [57], [60]	
	Input [28], [30], [34], [35], [37], [38] [42], [45], [47], [50]–[53] [54], [55], [57], [58], [60]–[62]	Output [28], [38], [46], [52], [53]	State [28], [33], [37], [39], [40] [42], [44]–[48], [50] [52]–[54], [56], [59]	

Example (II)

- Algorithm
 - In order to make a proposal, each agent calculates the optimal control action for a (sub)set of inputs that affect its dynamics

$$\{U_j^p(t)\}_{j \in n_p} = \arg \min_{\{U_j\}_{j \in n_p}} J_p(x_p, \{U_j\}_{j \in n_p})$$

s.t.

$$x_{p,k+1} = A_p x_{p,k} + \sum_{j \in n_p} B_{pj} u_{j,k}$$

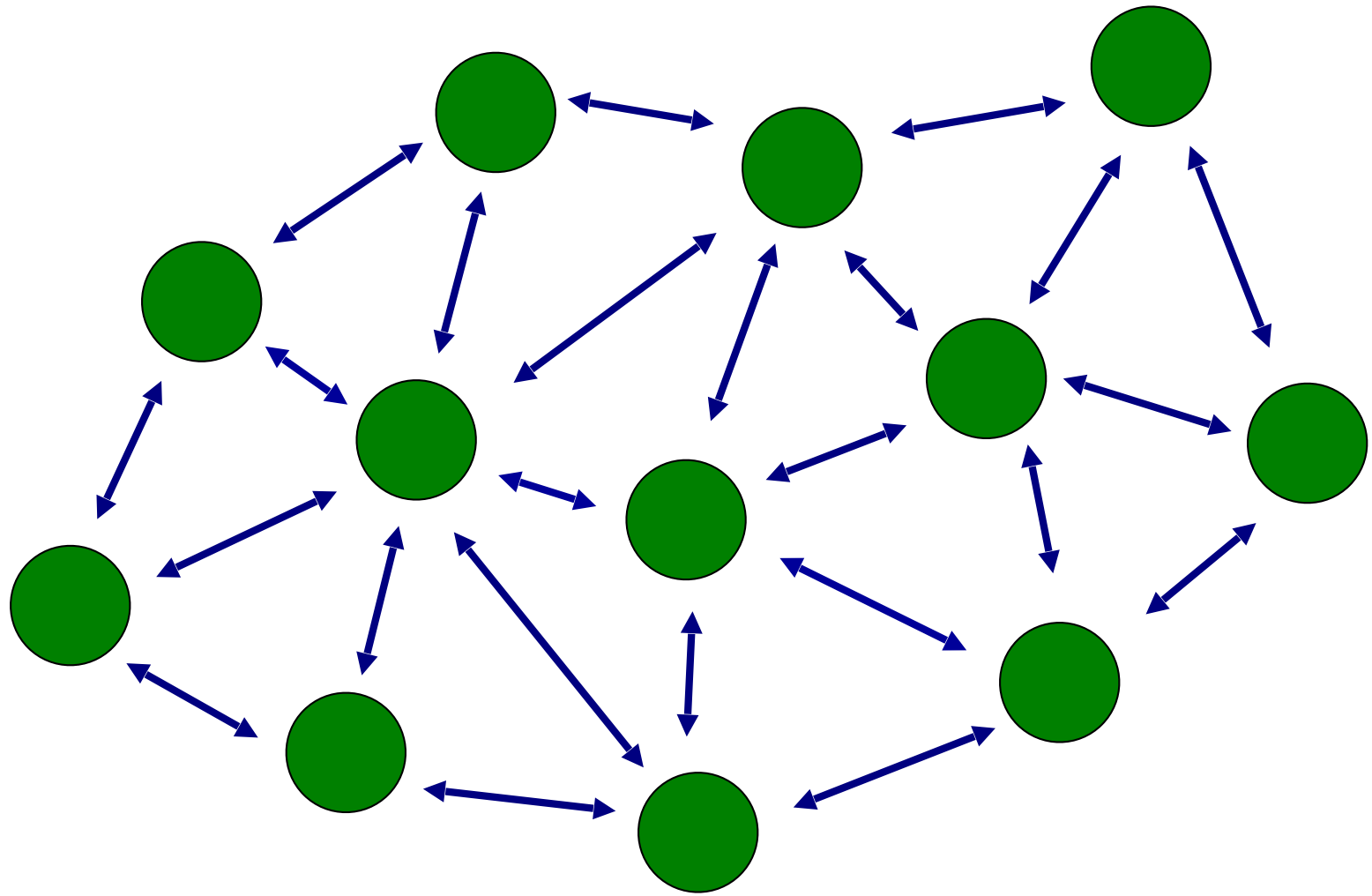
$$x_{p,0} = x_i(t)$$

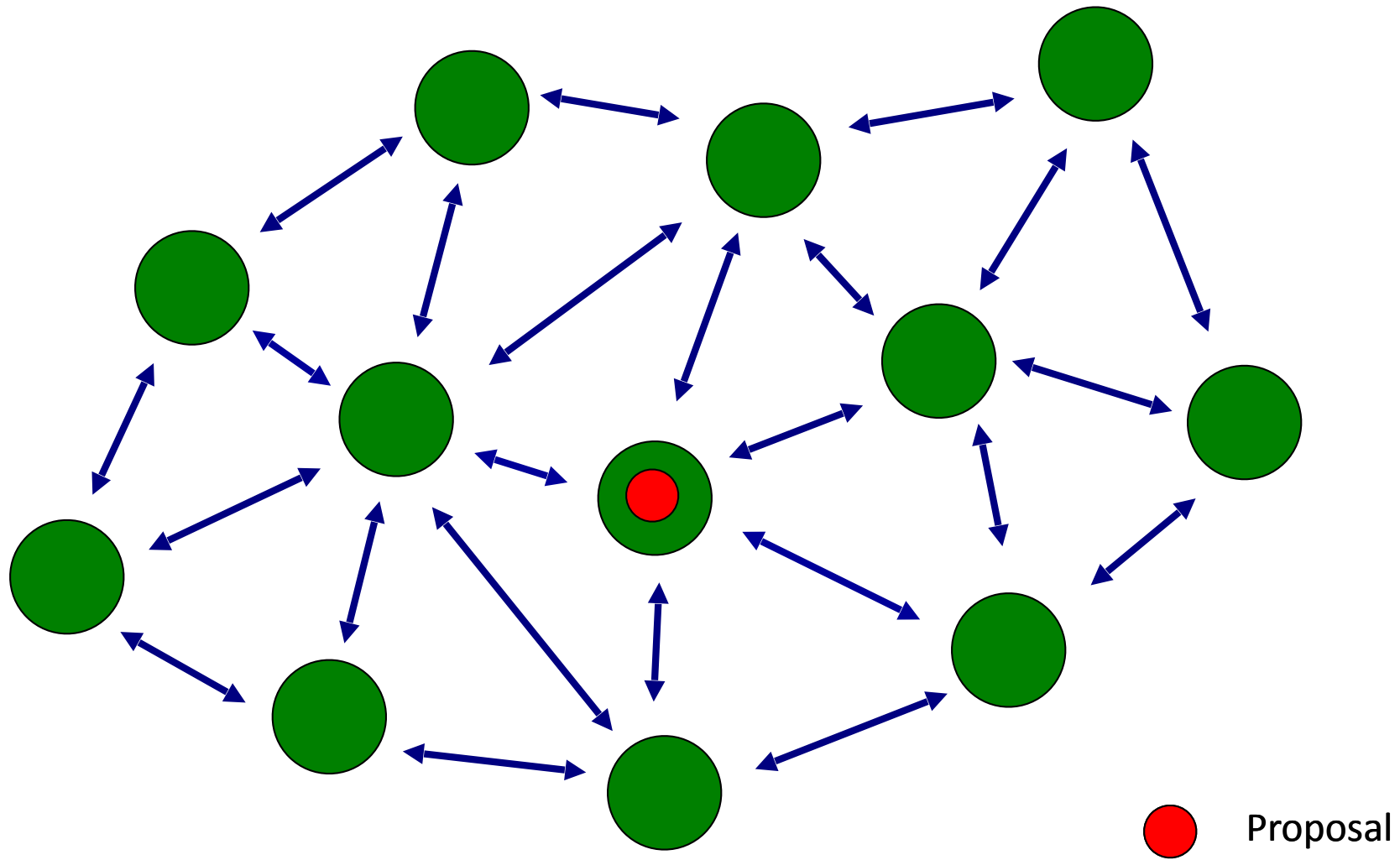
$$x_{p,k} \in \mathcal{X}_p, \quad k = 0, \dots, N$$

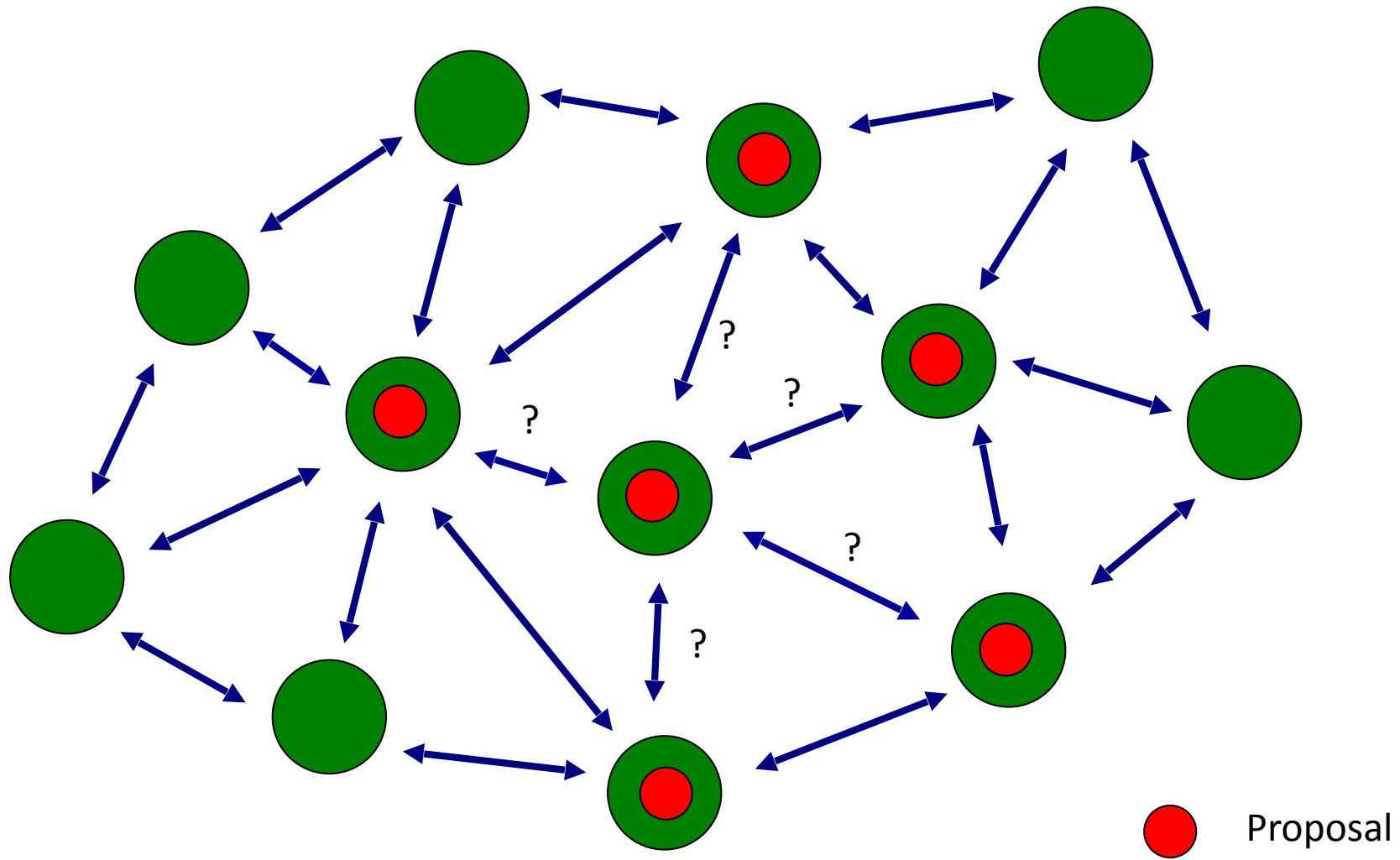
$$u_{j,k} \in \mathcal{U}_j, \quad k = 0, \dots, N - 1, \quad \forall j \in n_p$$

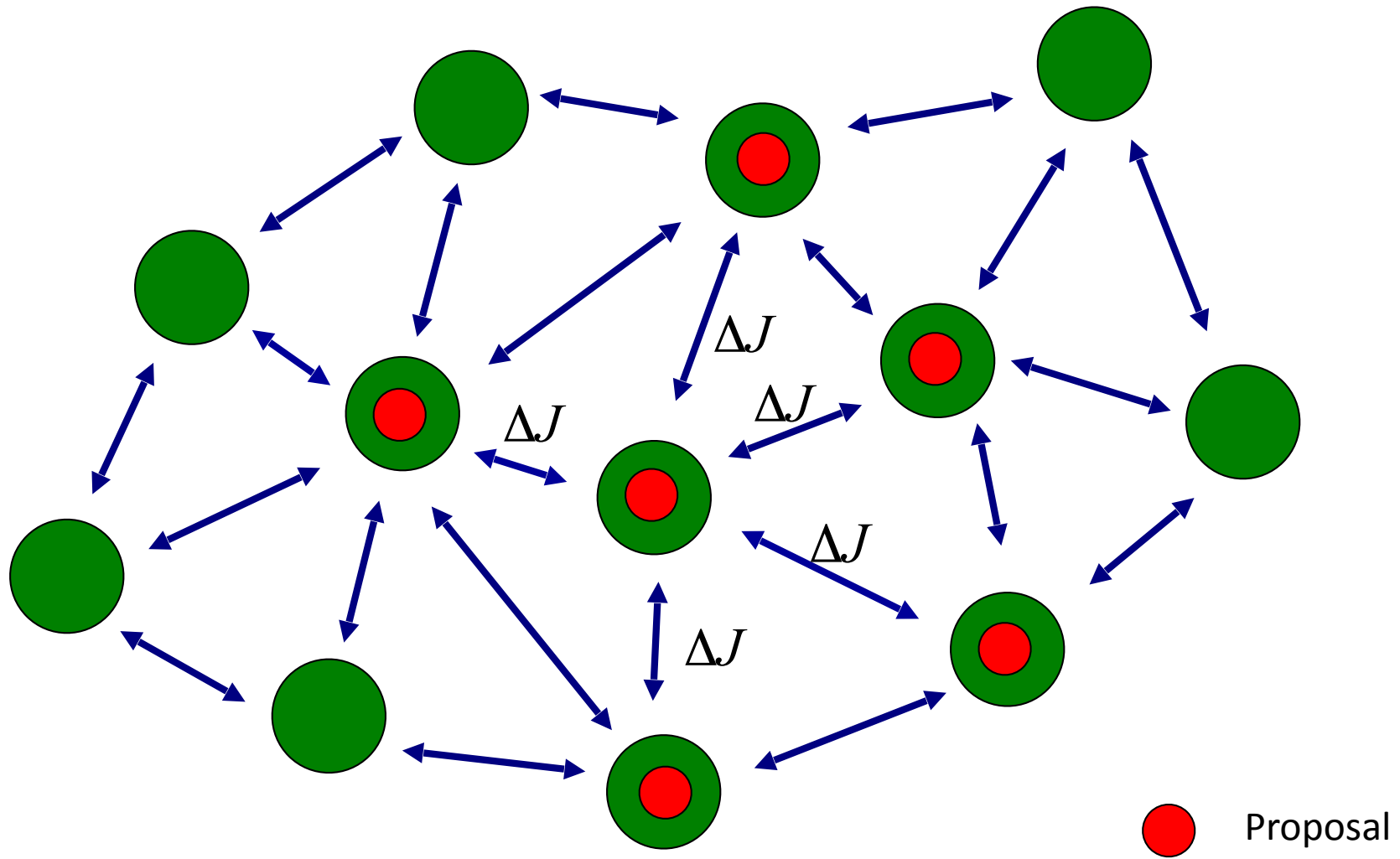
$$x_{p,N} \in \Omega_p$$

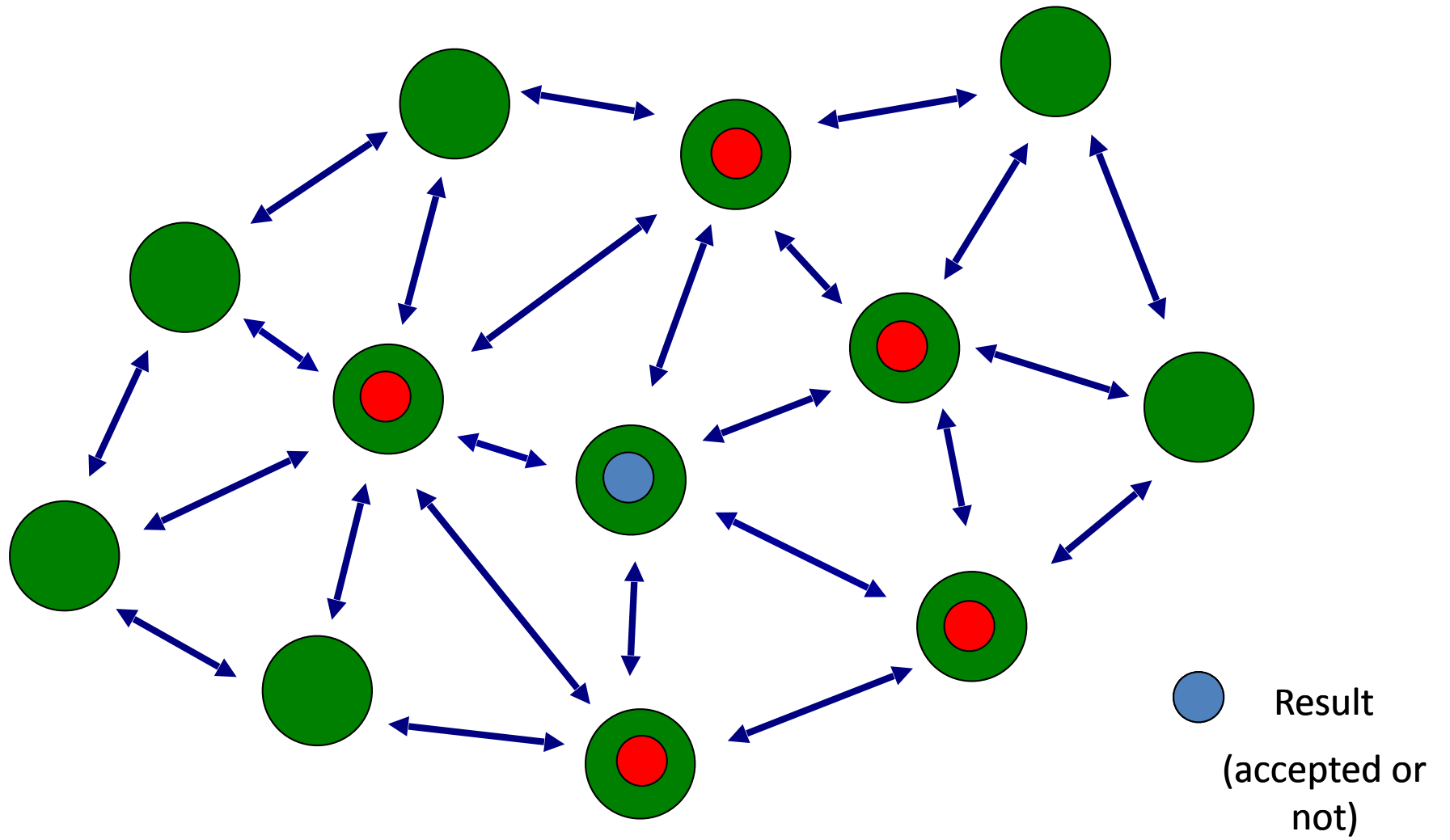
$$U_j = U_j^s(t), \quad \forall j \notin P_p$$

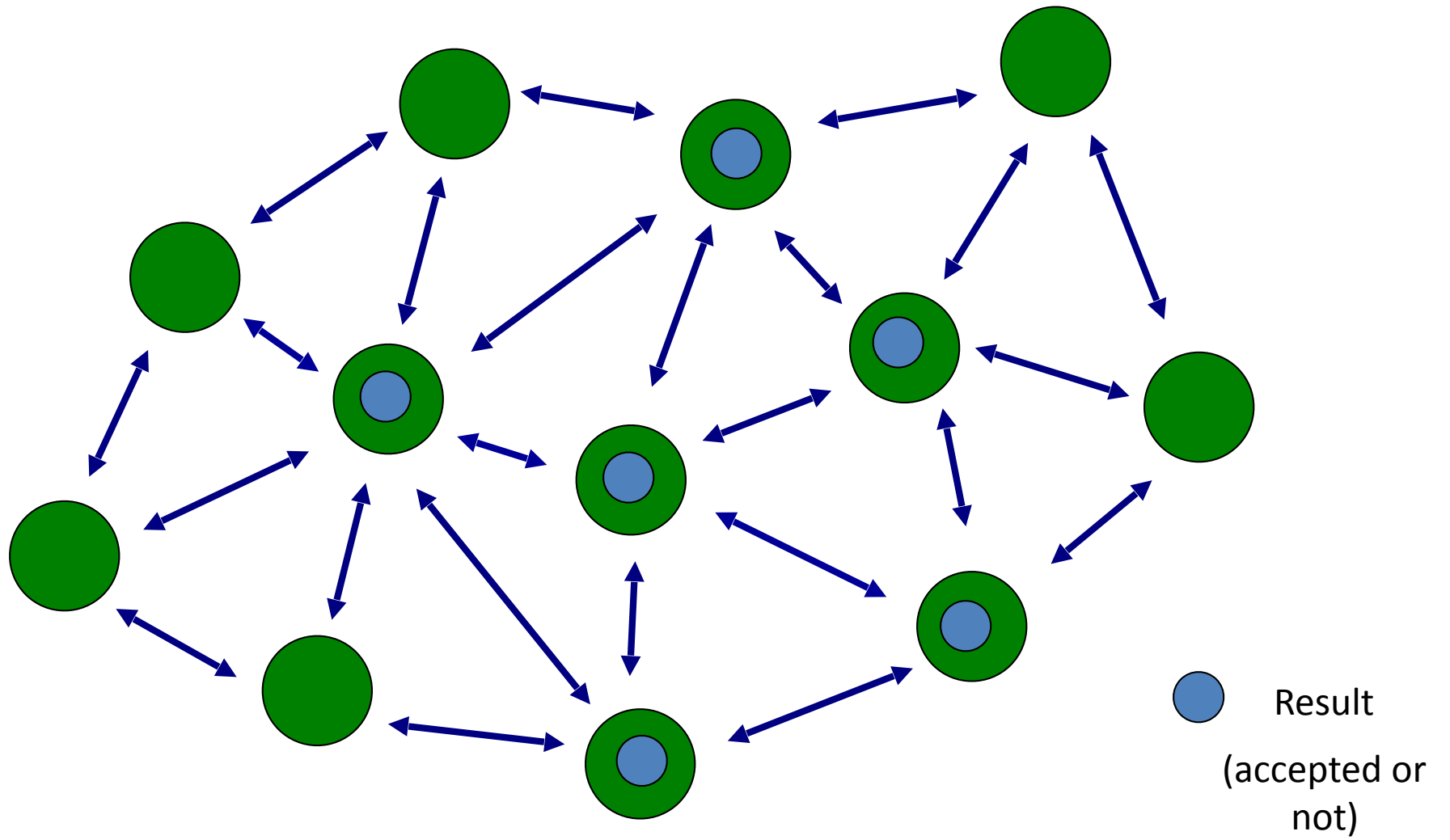




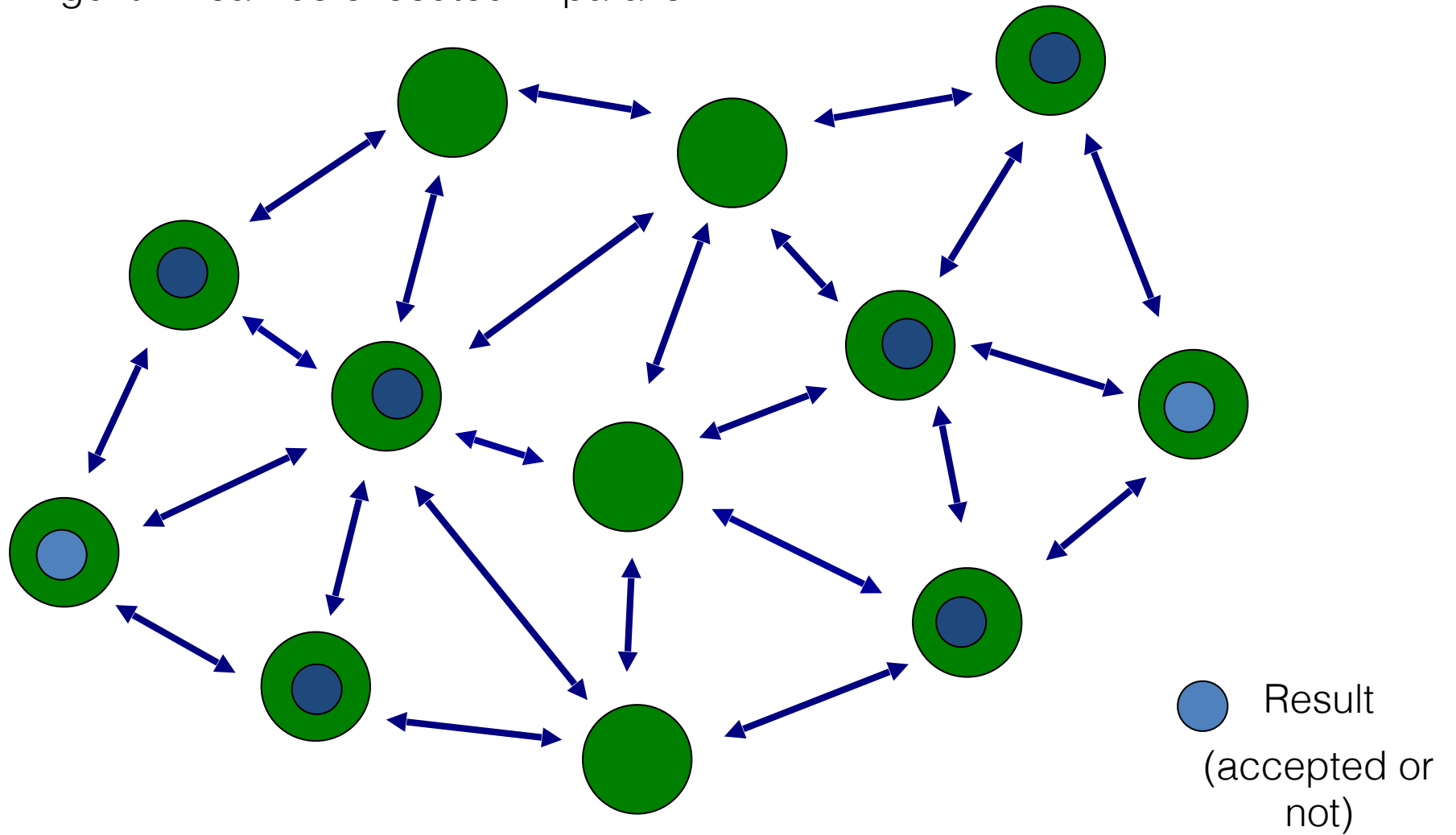








Algorithm can be executed in parallel



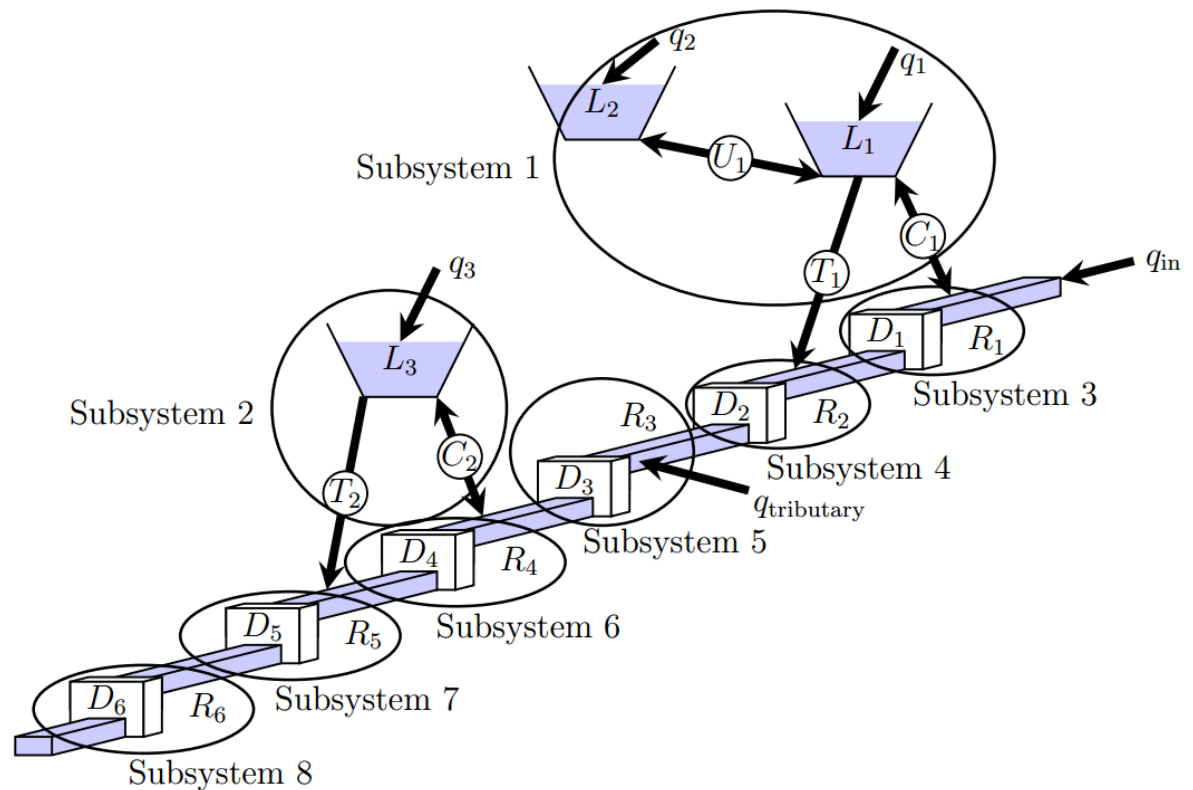
Example (II)

- Hydro-Power Valley (EDF)

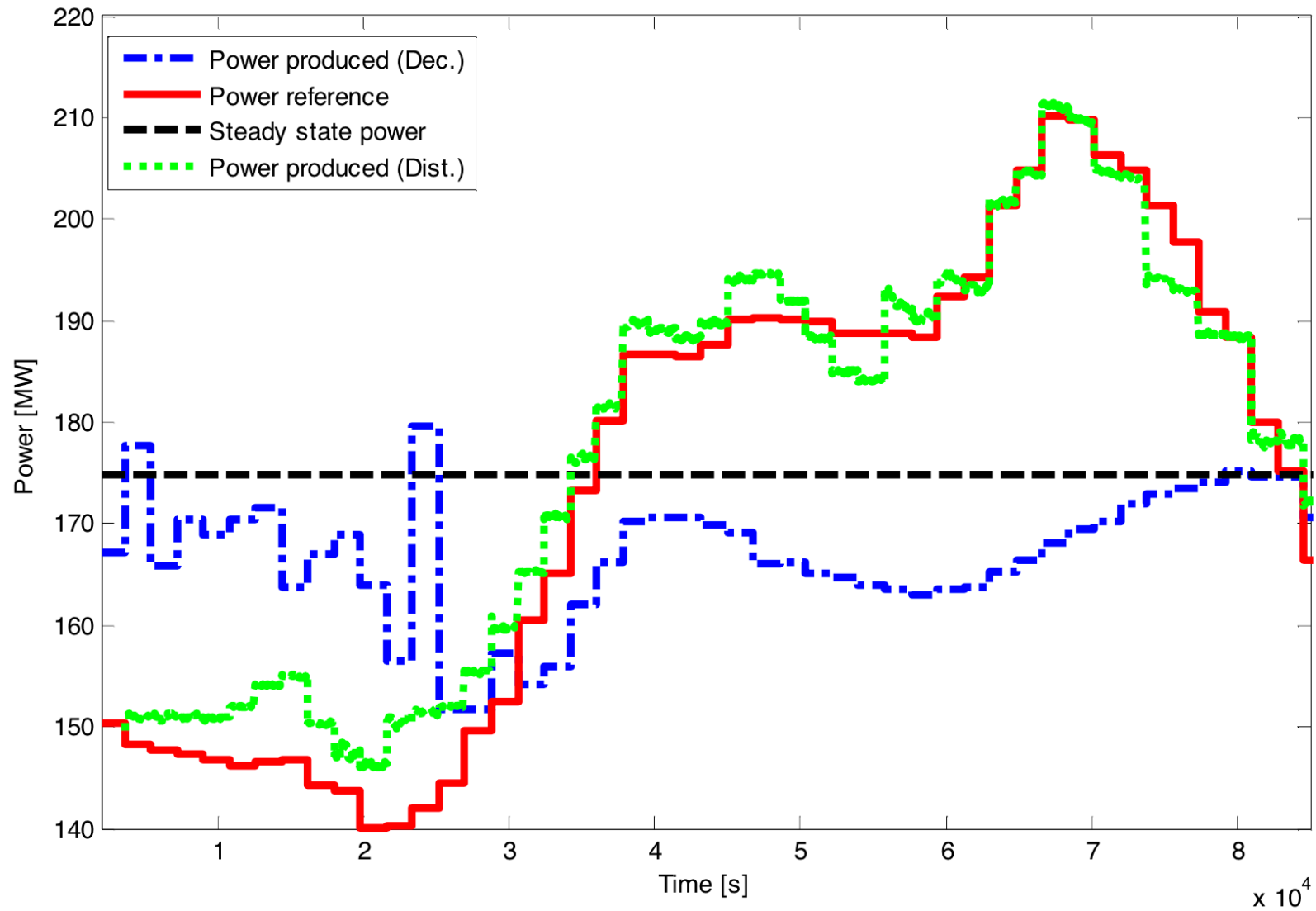
#8 subsystems

#249 states

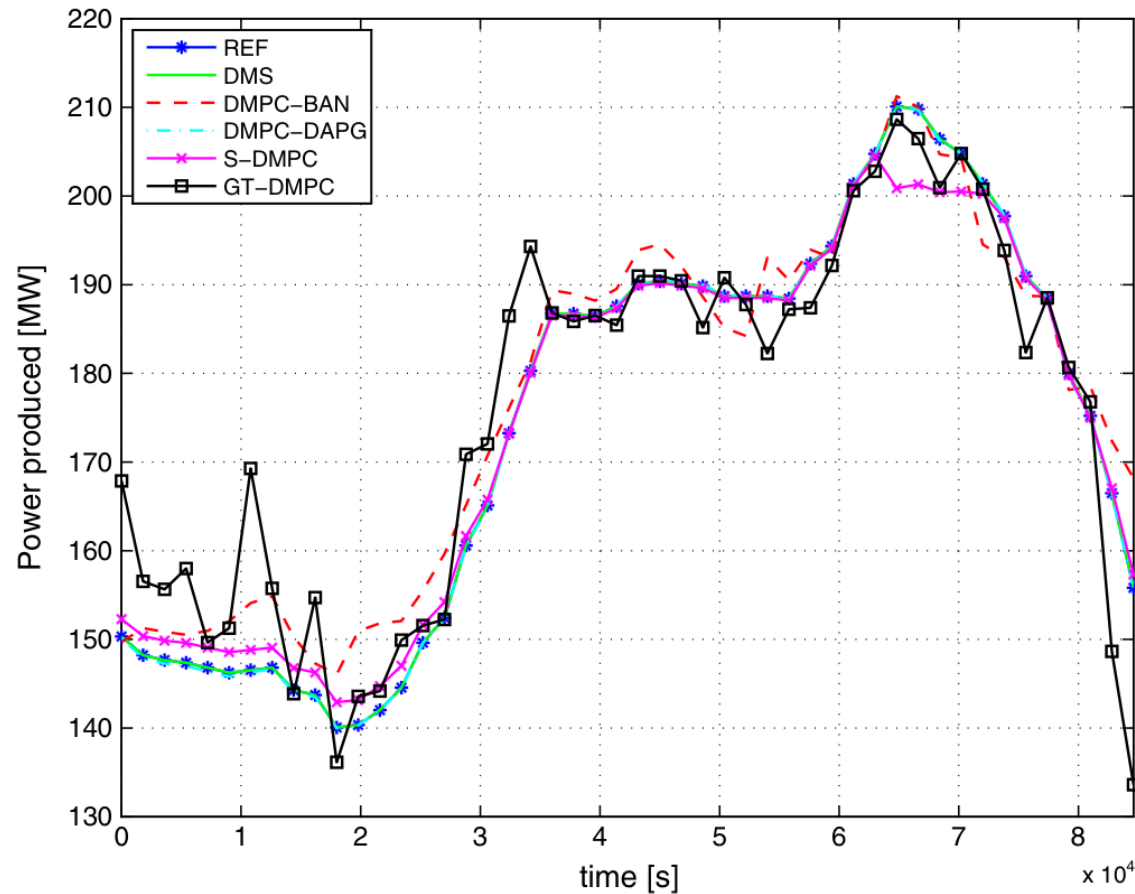
#10 inputs



Example (II)



Scheme	IATE (MW·h)	ETE (€)	ETE2 (€)	Comm. Cost. (#floats/sample]	Violations (m-h)	SAIDS (m ³ /s)	SAII (m ³ /s)
DMS	0.06	4	3	6 027 168	0.02	12 637.91	10 489.28
DMPC-BAN	89.09	5808	3388	1500	0.13	8152.11	2915.82
DAPG	3.86	250	229	1 218 000	0.62	7434.22	1034.83
S-DMPC	36.72	2419	1788	2937	2.10	8951.34	2731.54
GT-DMPC	116.39	7998	5184	500	60.48	11 267.17	5517.10



Maestre, J. M., Ridao, M. A., Kozma, A., Savorgnan, C., Diehl, M., Doan, M. D., Sadowska, A., Keviczky, T., De Schutter, B., Scheu, H., Marquardt, W., Valencia, F., and Espinosa, J. (2015), A comparison of distributed MPC schemes on a hydro-power plant benchmark. *Optim. Control Appl. Meth.*, 36, 306–332.

Distributed Model Predictive Control

Fallacies of Distributed Computing (1994)

- 1 The network is reliable.
- 2 Latency is zero.
- 3 Bandwidth is infinite.
- 4 The network is secure.
- 5 Topology doesn't change.
- 6 There is one administrator.
- 7 Transport cost is zero.
- 8 The network is homogeneous.

Distributed Model Predictive Control

□ DMPC based on dual decomposition

- We have a coupled optimization problem and we would like to solve it in a distributed fashion

$$J(U) = J_1(U) + J_2(U)$$

- Can we do this? $\min J(U) \stackrel{?}{\Leftrightarrow} \min J_1(U), \min J_2(U)$

Distributed Model Predictive Control

- The dual decomposition trick
 - We use auxiliary variables

$$\min J(U) \stackrel{?}{\Leftrightarrow} \begin{cases} \min J_1(U_1), \min J_2(U_2) \\ \text{s.t.} \\ U_1 = U_2 \end{cases}$$

- But how do we satisfy these new constraints in a distributed fashion?

Distributed Model Predictive Control

The dual decomposition trick

- We use Lagrangian prices

$$\left. \begin{array}{l} \min_{U_1, U_2} J_1(U_1) + J_2(U_2) \\ \text{s.t.} \\ U_1 = U_2 \end{array} \right\} \Leftrightarrow \max_{\lambda} \min_{U_1, U_2} J_1(U_1) + J_2(U_2) + \lambda(U_1 - U_2)$$

- There is an incentive to make $U_1=U_2$ to minimize costs
- Can we solve this now in a distributed fashion?

YES!!!

Distributed Model Predictive Control

The dual decomposition trick

- At time k , we start with initial prices $\lambda_0(k)$
- Each local controller optimizes a local cost function

$$\min_{U_1} J_1(U_1) + \lambda U_1 \qquad \min_{U_2} J_2(U_2) - \lambda U_2$$

- After local optimization, prices are updated by a coordinator following a gradient method in order to reduce the constraint violation until convergence is attained

$$\lambda_{l+1}(k) = \lambda_l(k) + \gamma(U_1 - U_2)$$

Distributed Model Predictive Control

What happens if agent 1 solves a different problem?

- For example, by using different prices

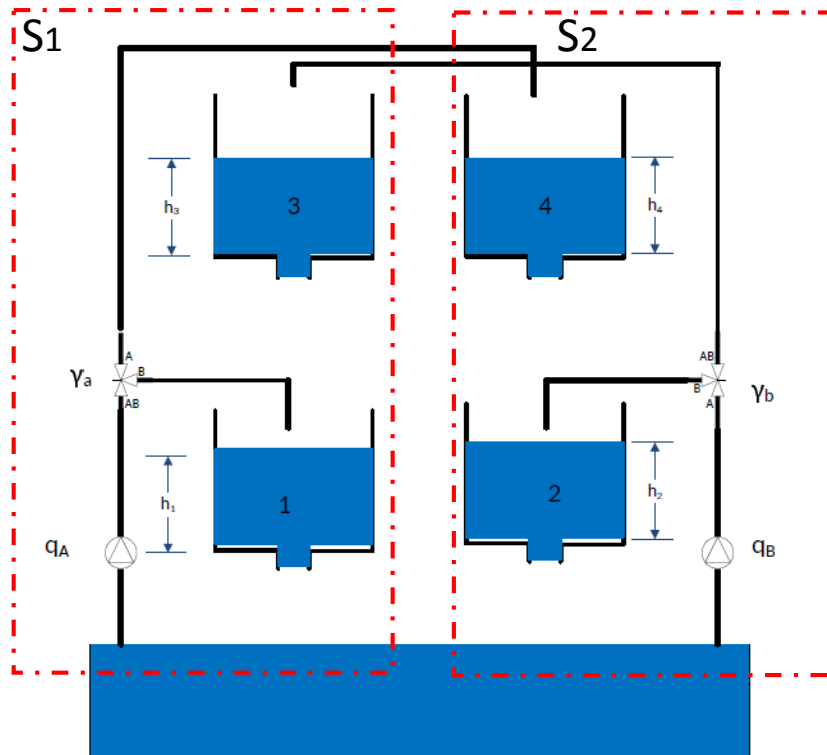
$$\min_{U_1} J_1(U_1) + \frac{\lambda}{\alpha} U_1$$

- Or by introducing fake constraints or implementing a different control action



Velarde, P., Maestre, J. M., Ishii, H., & Negenborn, R. R. Vulnerabilities in Lagrange-based distributed model predictive control. Optimal Control Applications and Methods. In press. [Available on line]

Example (III)



Objective Function

$$J(x[k], u[k]) = \min_{u[k:k+N_p-1]} \sum_{i=k}^{k+N_p} (x[i] - x_{\text{ref}})^T Q (x[i] - x_{\text{ref}}) + u^T[i] R u[i]$$

System Model

$$\frac{dh_1}{dt} = -\frac{a_1}{A_1} \sqrt{2gh_1} + \frac{a_3}{A_3} \sqrt{2gh_3} + \frac{\gamma_a}{A_1} q_A,$$

$$\frac{dh_2}{dt} = -\frac{a_2}{A_2} \sqrt{2gh_2} + \frac{a_4}{A_4} \sqrt{2gh_4} + \frac{\gamma_b}{A_2} q_B,$$

$$\frac{dh_3}{dt} = -\frac{a_3}{A_3} \sqrt{2gh_3} + \frac{(1 - \gamma_b)}{A_3} q_B,$$

$$\frac{dh_4}{dt} = -\frac{a_4}{A_4} \sqrt{2gh_4} + \frac{(1 - \gamma_a)}{A_4} q_A,$$

Constraints

$$0.2 \text{ m} \leq h_1[k], h_3[k] \leq 1.36 \text{ m},$$

$$0.2 \text{ m} \leq h_2[k], h_4[k] \leq 1.36 \text{ m},$$

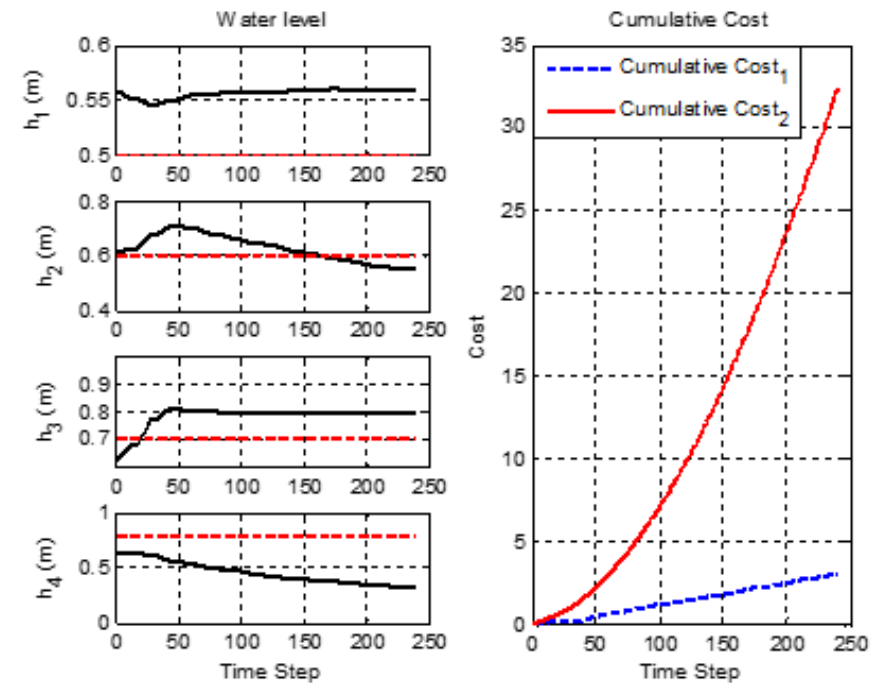
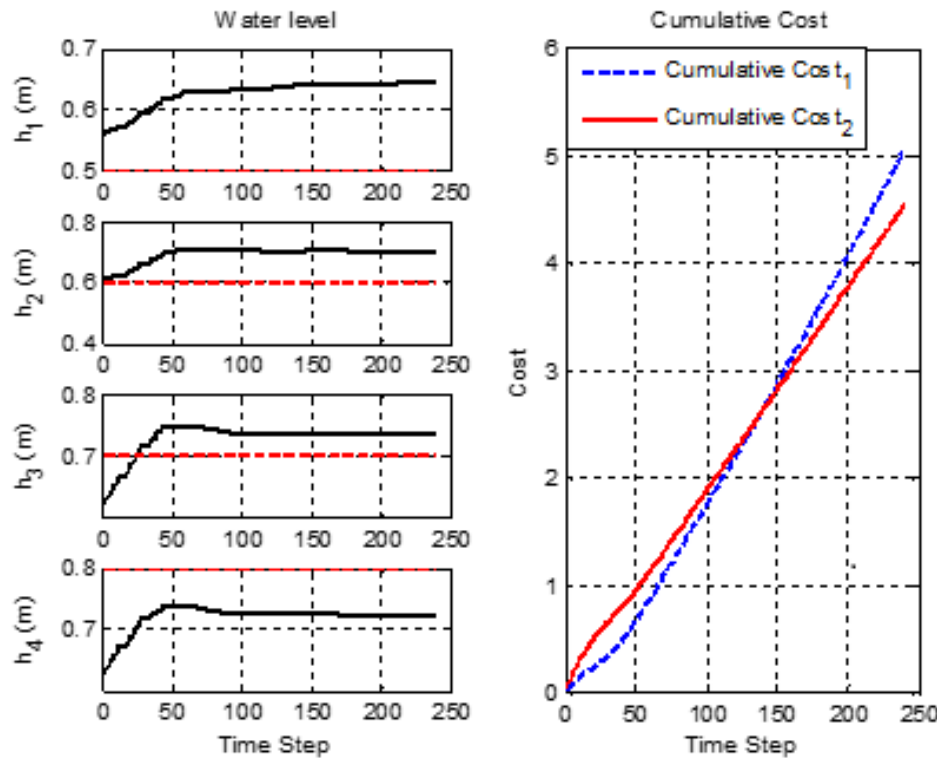
$$0 \text{ m}^3/\text{h} \leq q_A[k] \leq 3.26 \text{ m}^3/\text{h},$$

$$0 \text{ m}^3/\text{h} \leq q_B[k] \leq 4 \text{ m}^3/\text{h}.$$

Example (III)

Standard DMPC

Fake prices



$$\alpha = 10$$

Distributed Model Predictive Control

Goal:

Provide robustness to the subsystems in a distributed fashion.

Alternatives:

- MS-DMPC
- TB-DMPC
- Secure dual decomposition DMPC

Scenario Generation

Noise was added to the controllers' states $x_i[k]$ at each time step considered in the experiments.

$$\Lambda_i(k) = \{\lambda_i^1(k), \lambda_i^2(k), \dots, \lambda_i^{N_s}(k)\}.$$

Set of collected scenarios

MS-DMPC

$$\min_{u_i[k:k+N_p-1]} \sum_{l=1}^{N_s} \rho_i^l \sum_{j=k}^{k+N_p-1} (\ell_i(x_i^l(j+1), u_i(j)) + \lambda^l(j)^T C_i u_i(j)),$$

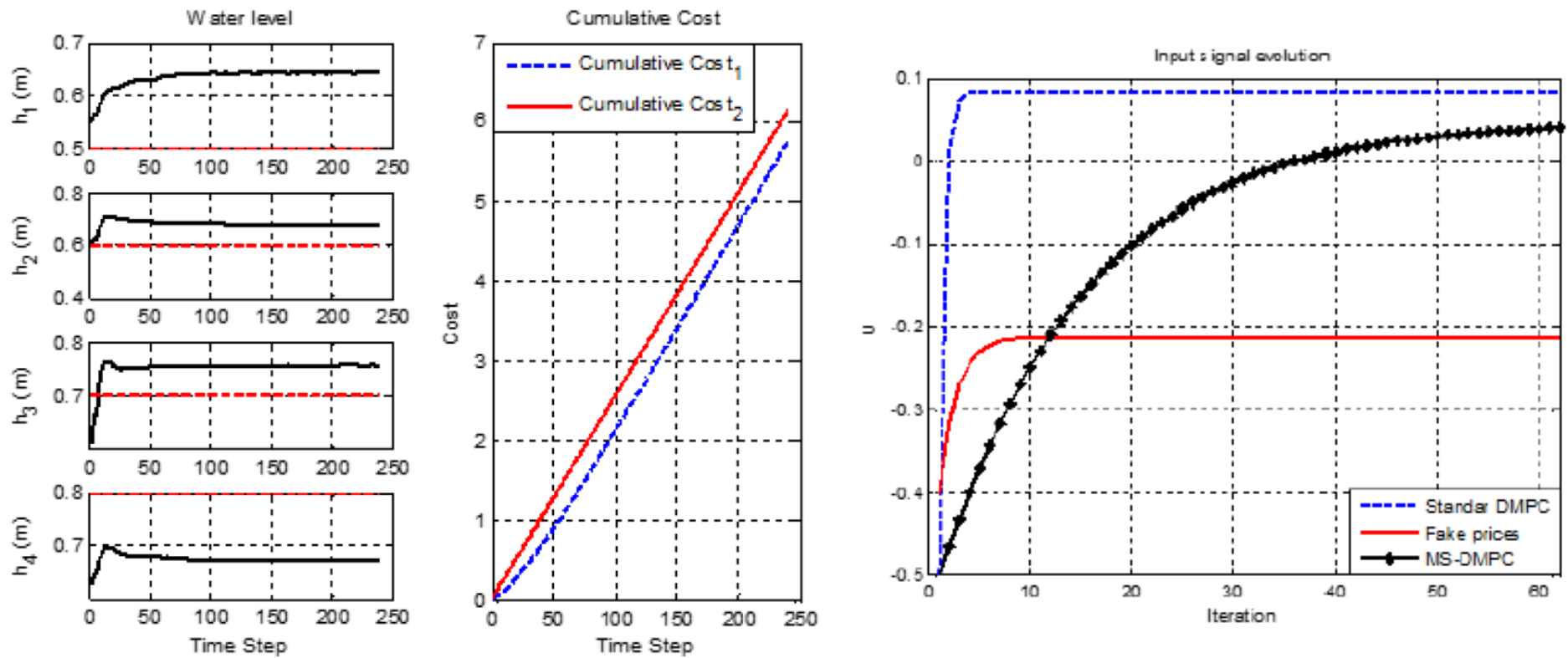
subject to

$$\begin{aligned} x_i^l(j+1) &= A_i^l x_i^l(j) + B_i u_i(j), \\ x_i^l(j) &\in \mathcal{X}_i, \quad \forall j \in \mathbb{Z}_+, \quad \forall l \in \mathbb{Z}_1^{N_s}, \\ u_i(j) &\in \mathcal{U}_i, \quad \forall j \in \mathbb{Z}_+, \quad \forall l \in \mathbb{Z}_1^{N_s}, \end{aligned}$$

where ρ^l is the probability of occurrence of each scenario l .

Example (III)

MS-MPC Defense Mechanism Running



Outline

- Model Predictive Control
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- **Coalitional Model Predictive Control**
- Conclusions

Coalitional Model Predictive Control

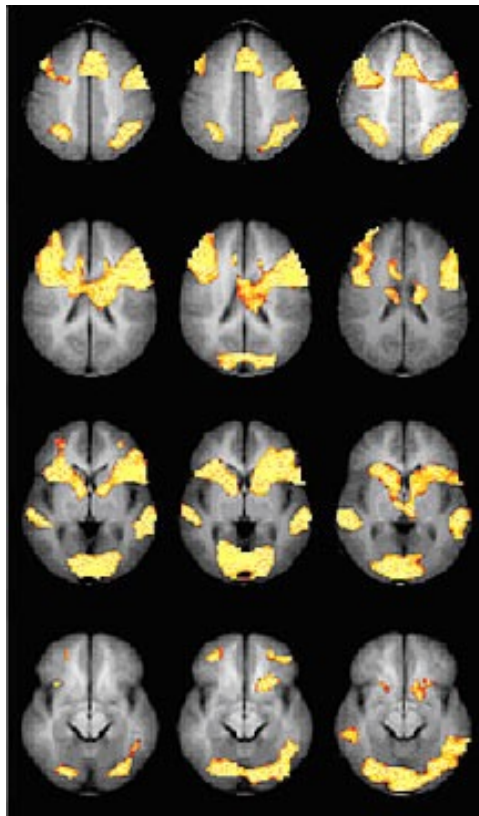
- Typical assumption
 - Coupling does not change with time
 - Consequently neighborhoods are static

But does coupling change with time?

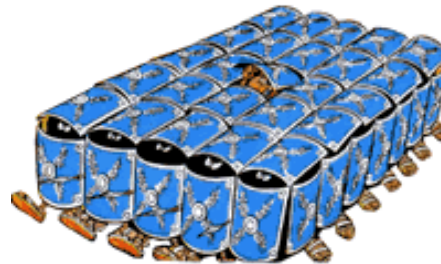
Coalitional Model Predictive Control

- It does. Coupling changes with time...

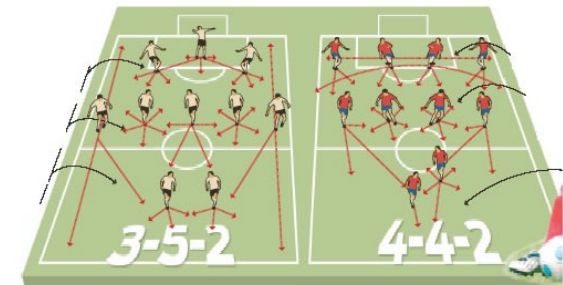
Neurons



Formations

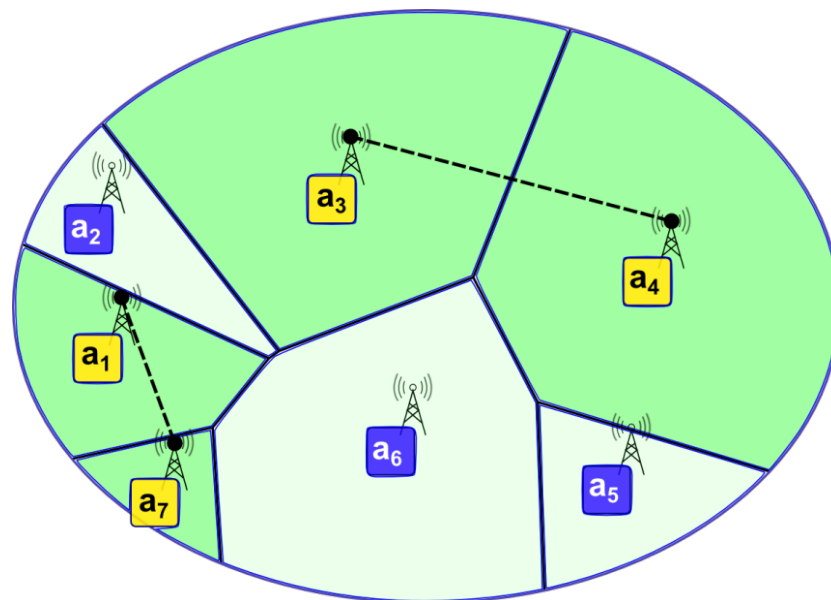


Strategies



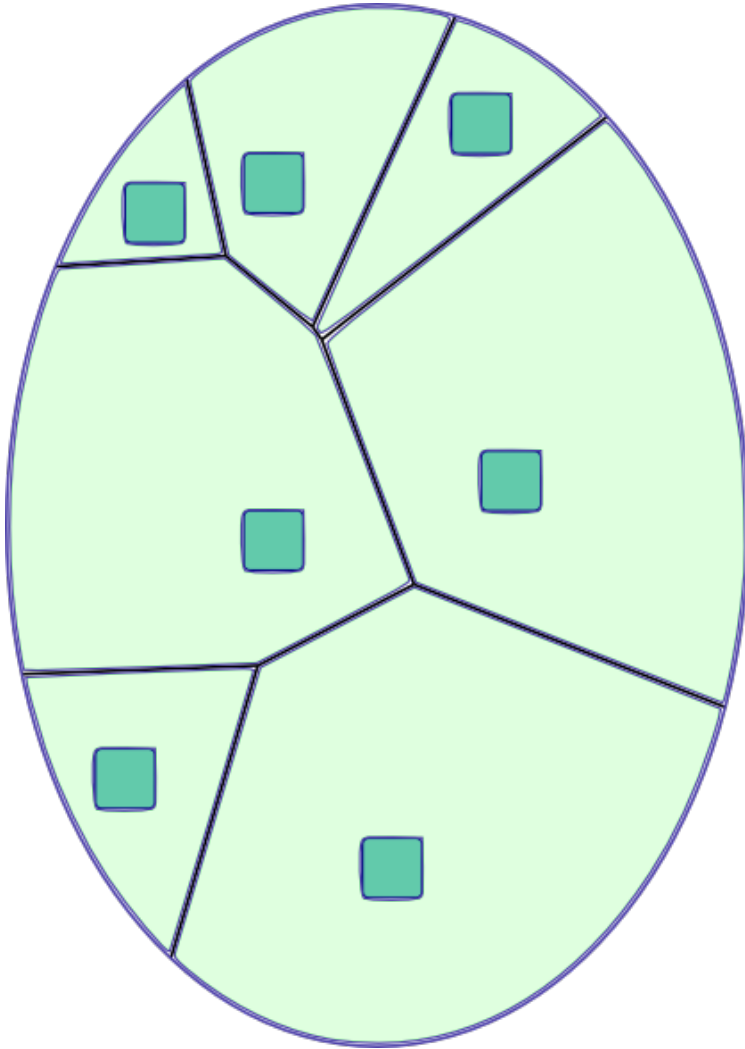
Coalitional Model Predictive Control

- Coalitions are groups of agents with relevant coupling at a given period of time
- Static coalitions already exist in control: partitions



Decentralized control

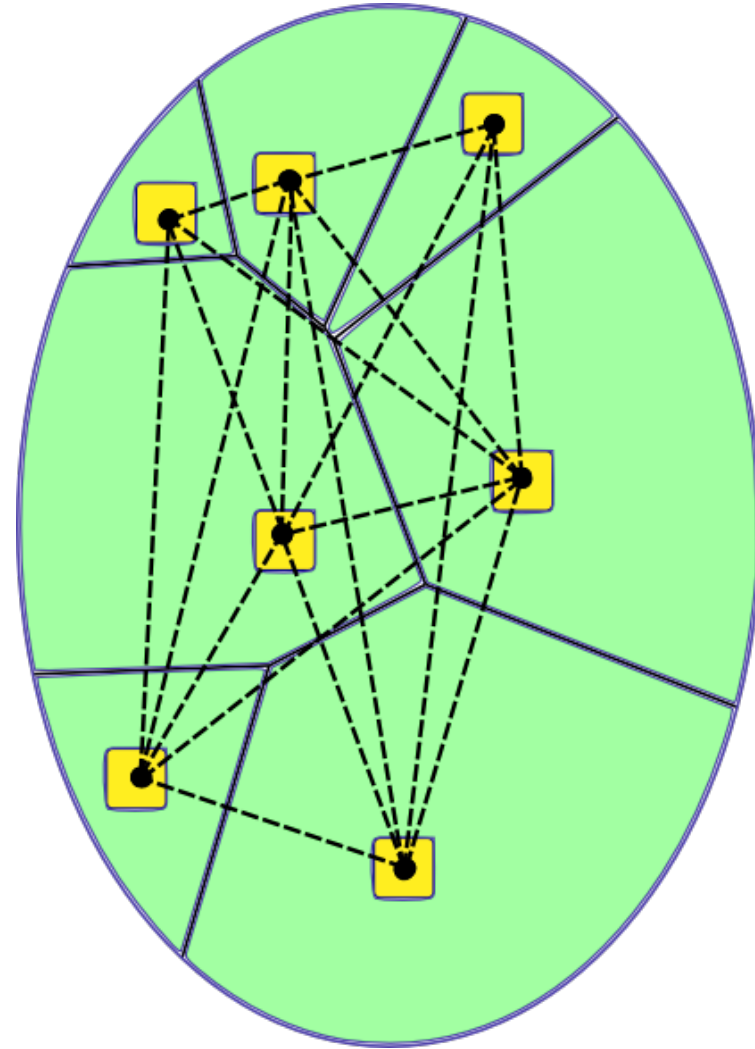
No information exchange
Mutual interactions are neglected



Limited coupling

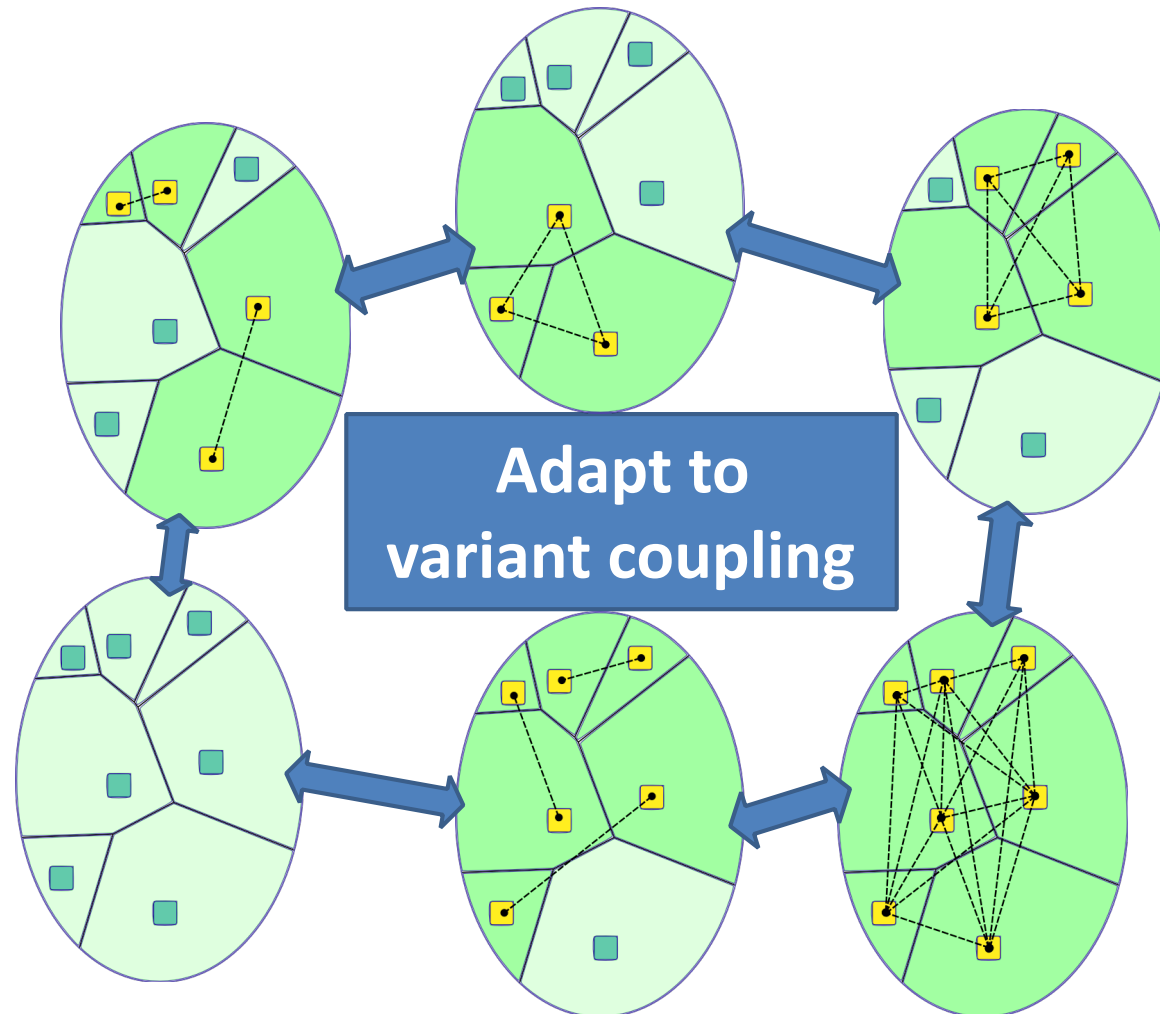
Distributed control

Continuous information exchange
Interactions are handled

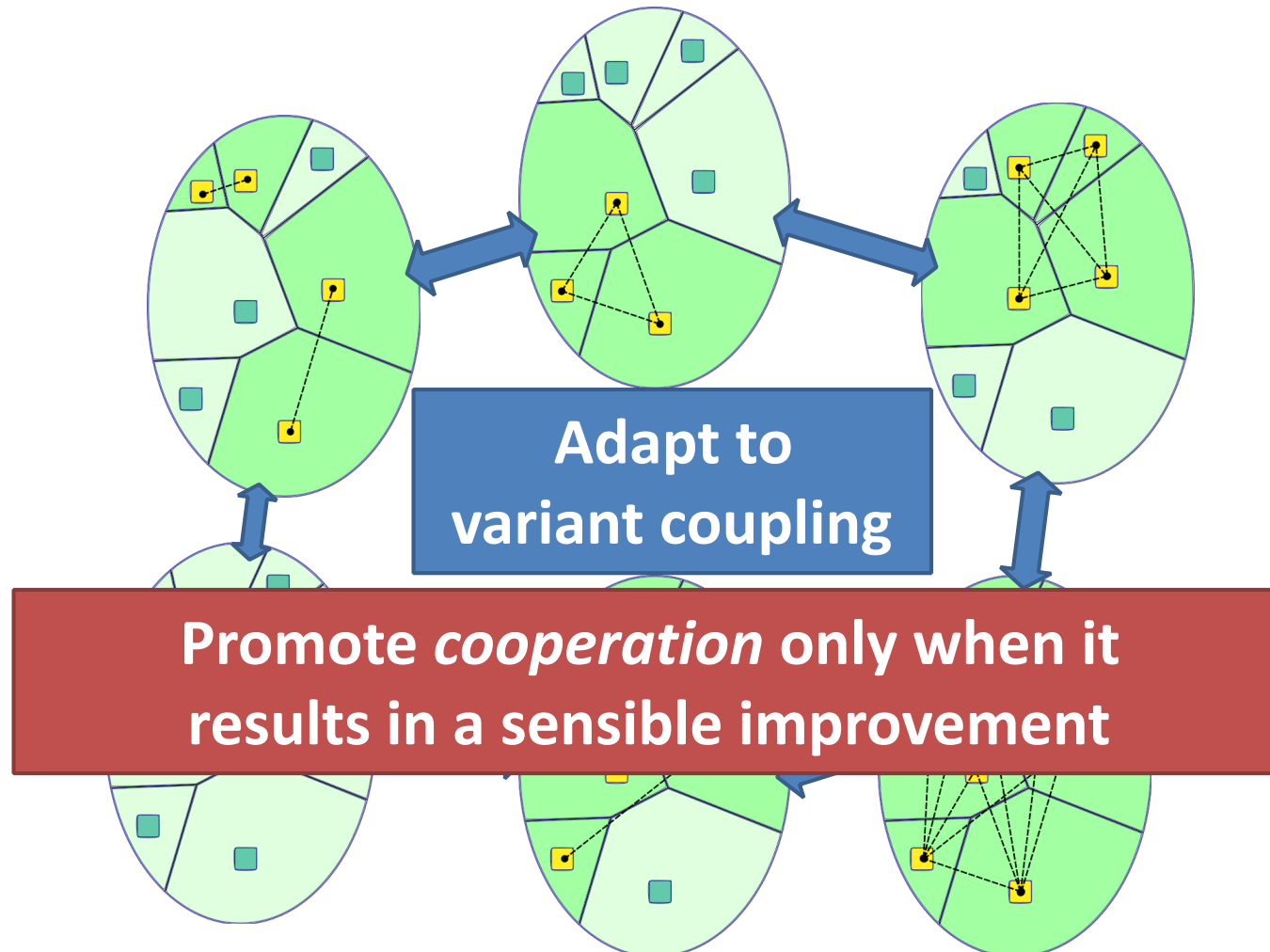


Strong coupling

Coalitional Model Predictive Control



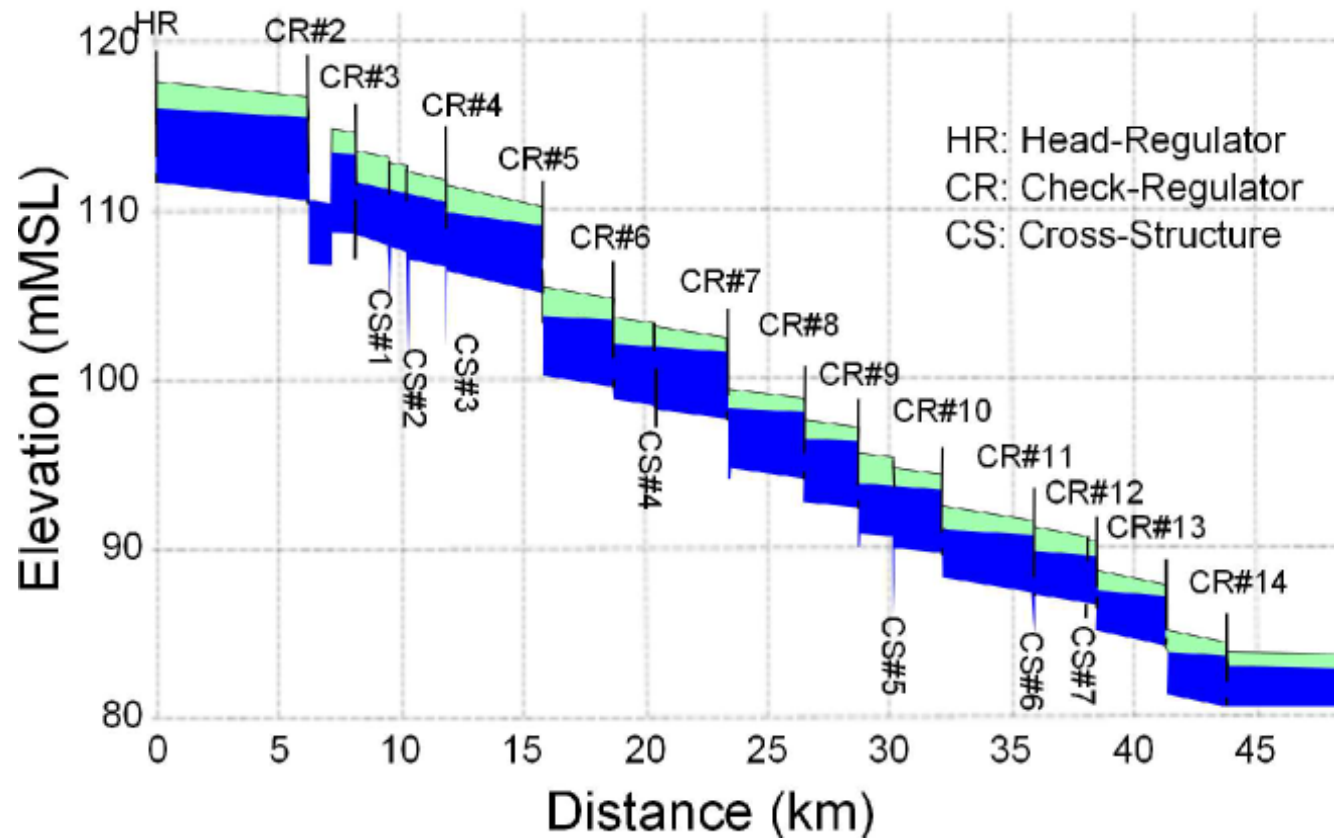
Coalitional Model Predictive Control



Coalitional Model Predictive Control

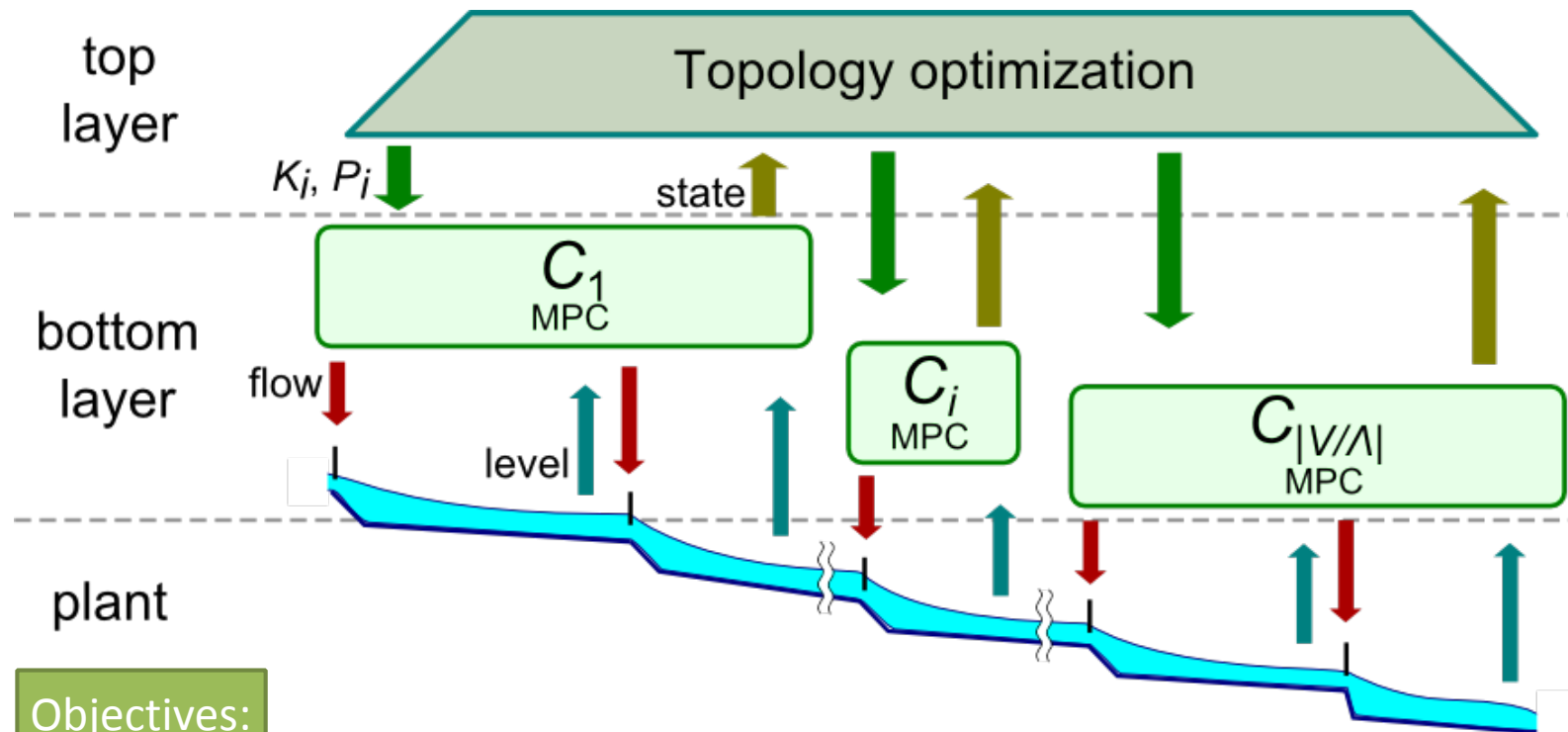
- We have developed different coalitional control algorithms that promote coalitional behavior
- Two approaches:
 - Top-down (Hierarchical)
 - Bottom-up

Example (IV)



Section of the Dez
main canal (Iran)

Example (IV)



Objectives:

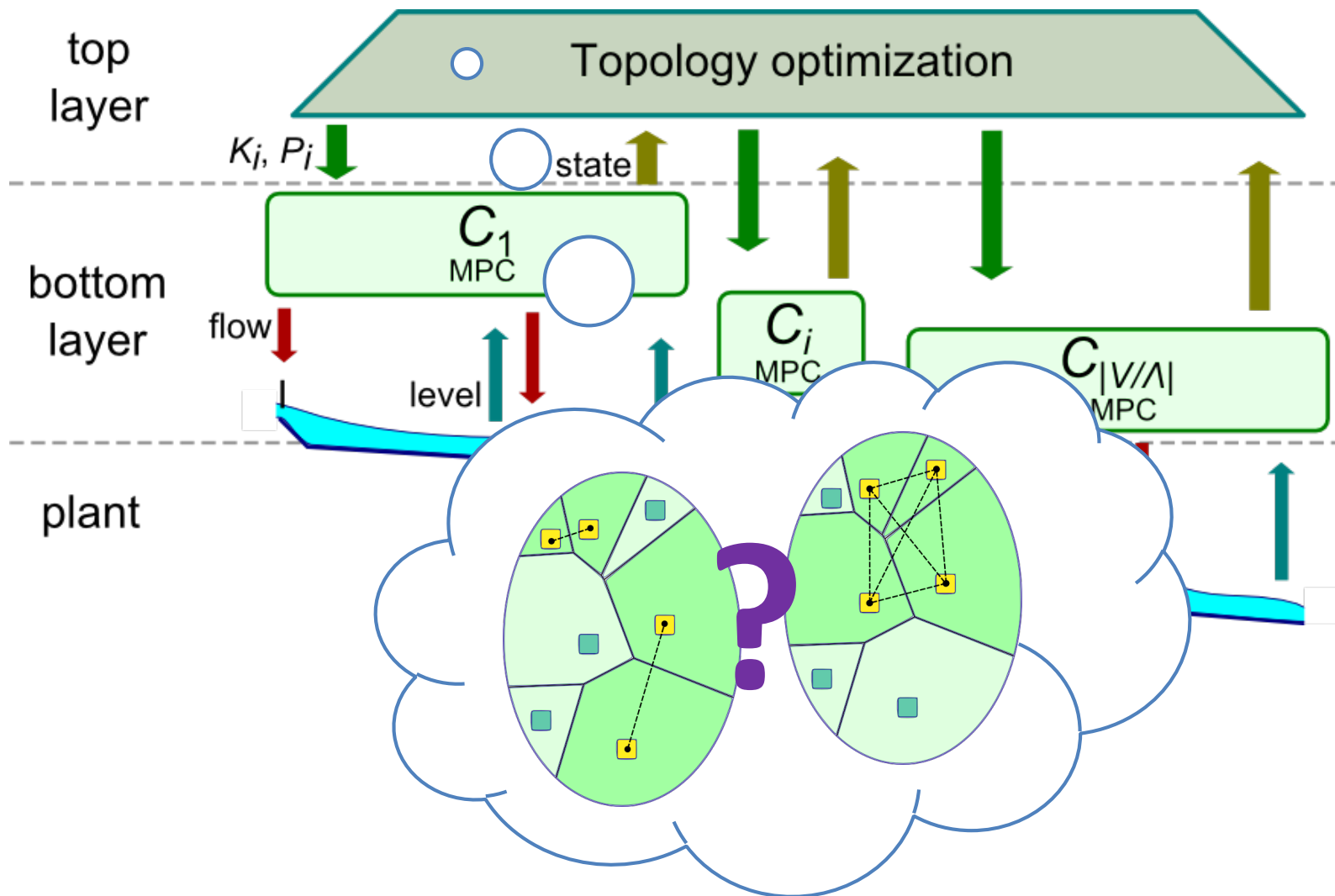
Regulate water levels

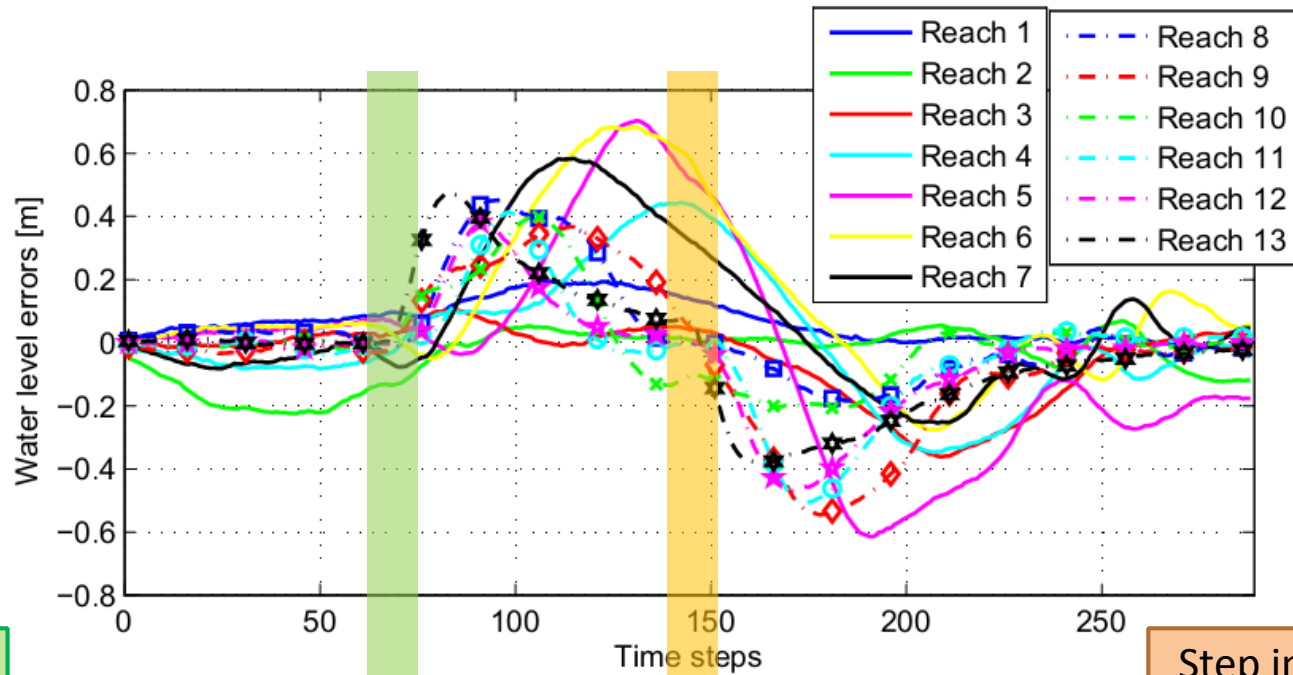
Compensate for disturbances represented by changes of the offtakes

Minimize the use of network resources

Example (IV)

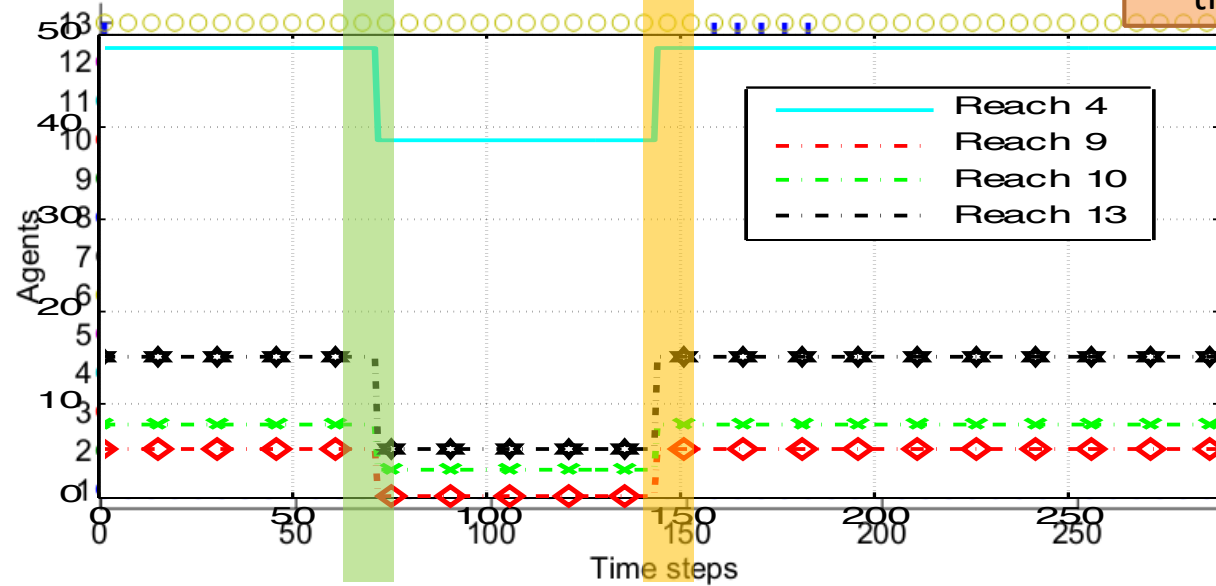
Improvement of performance through coalitional control

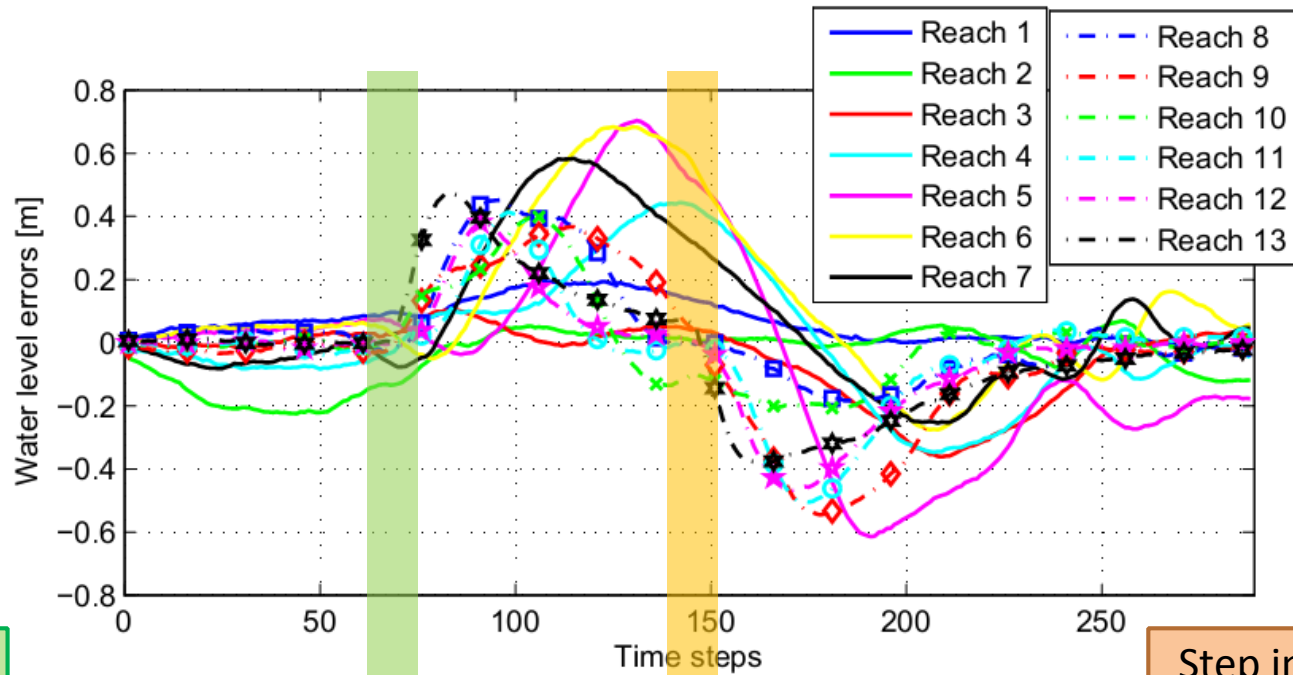




Step decrease in the offtakes

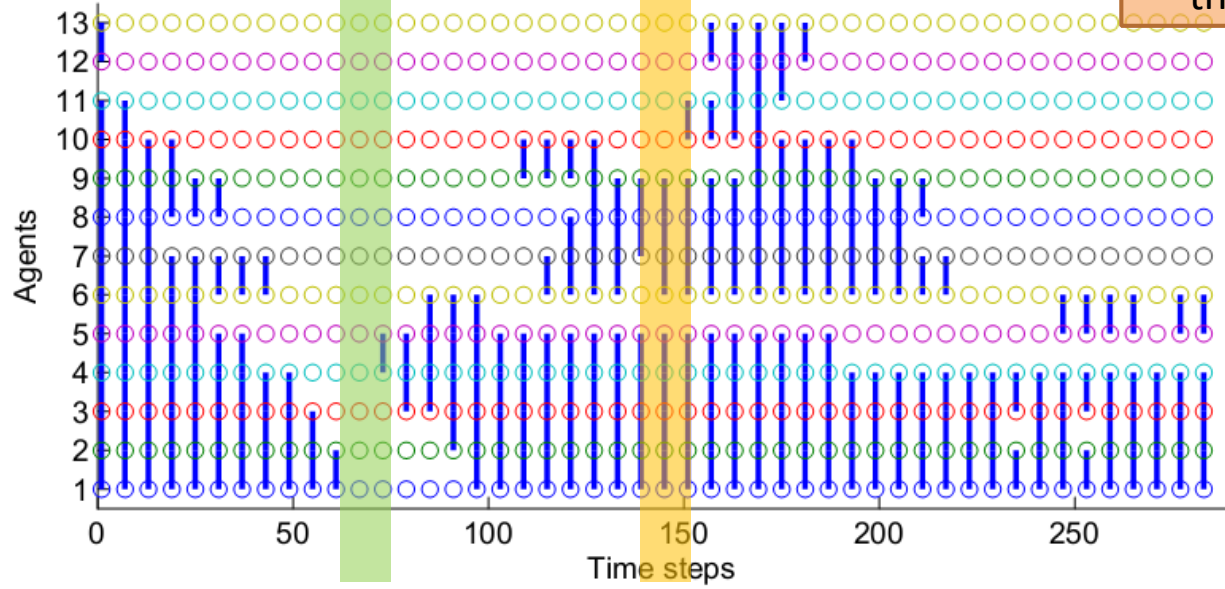
Step increase in the offtakes



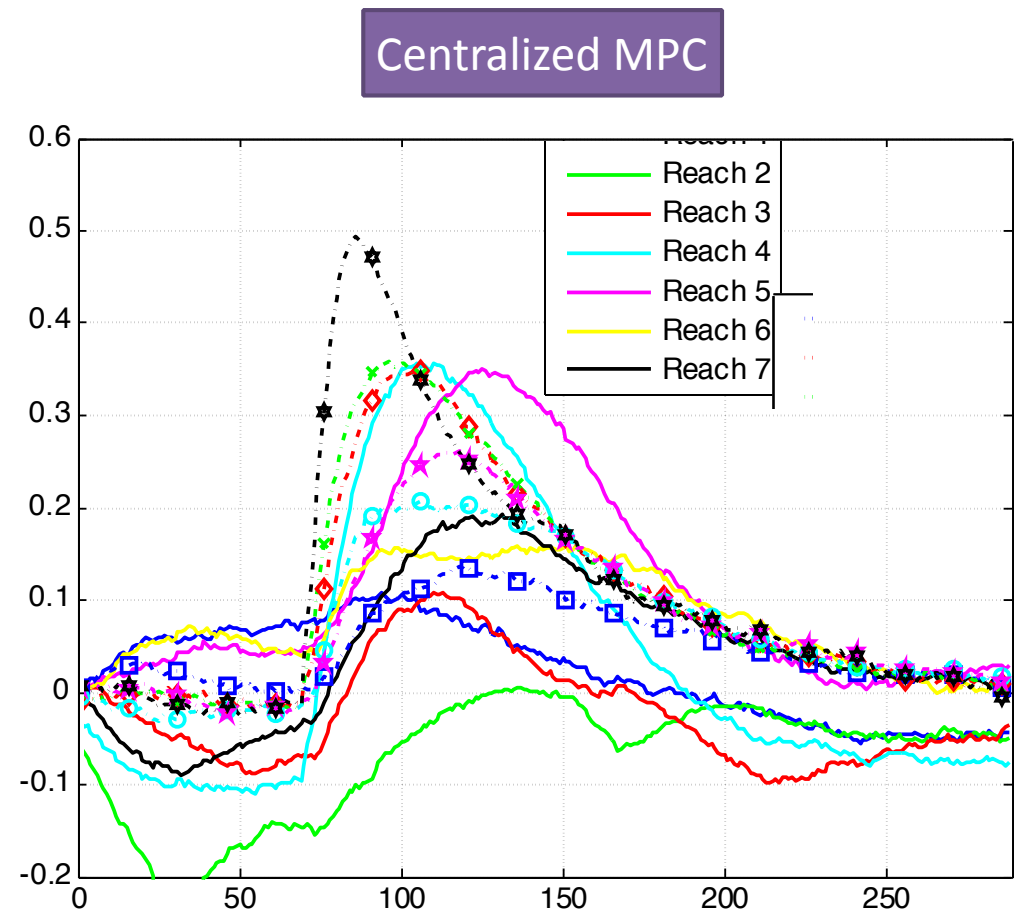
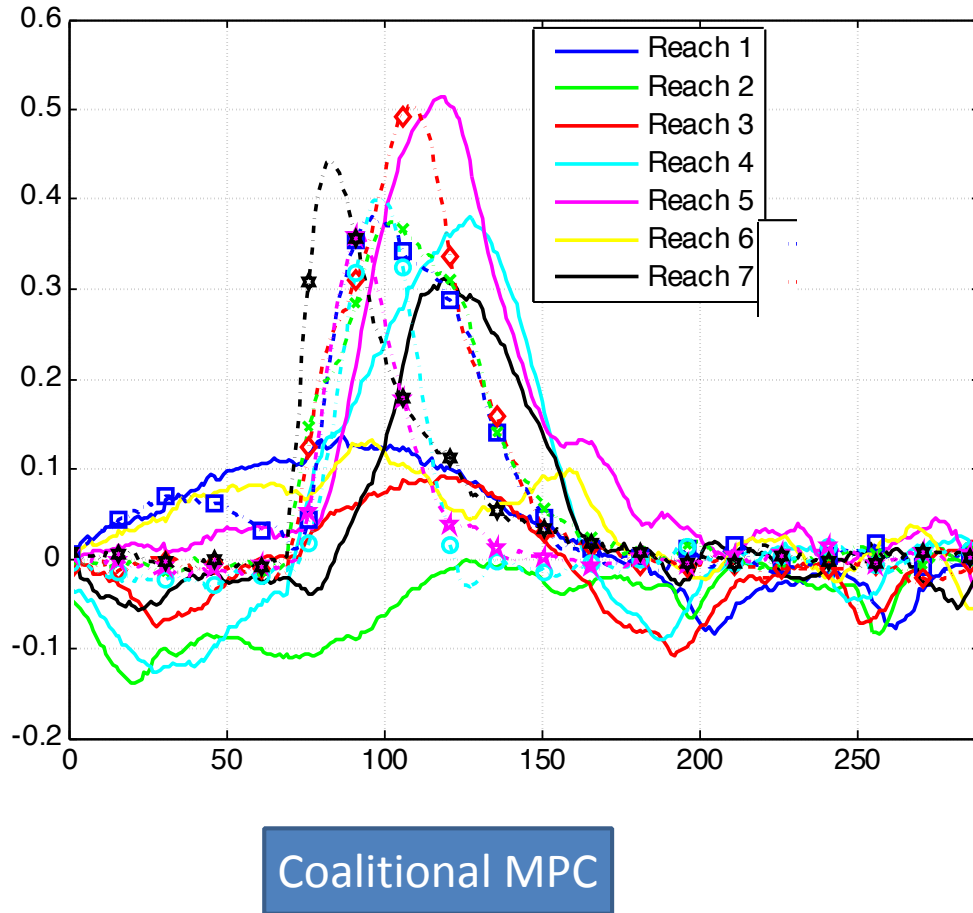


Step decrease in the offtakes

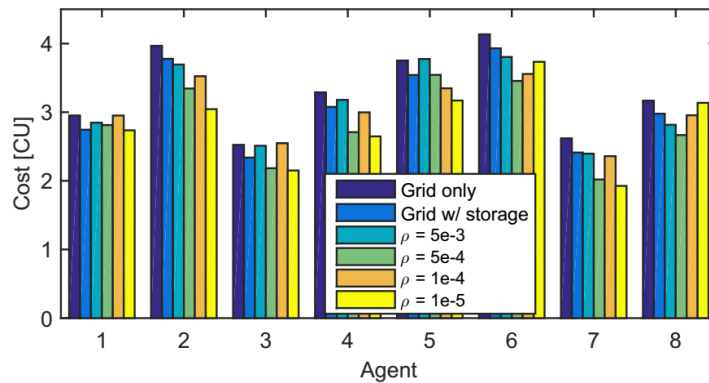
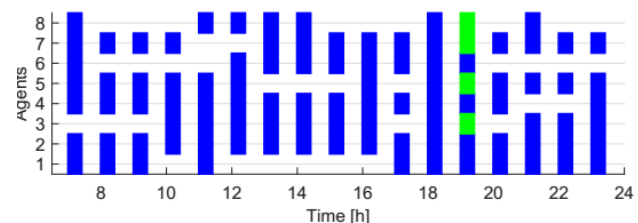
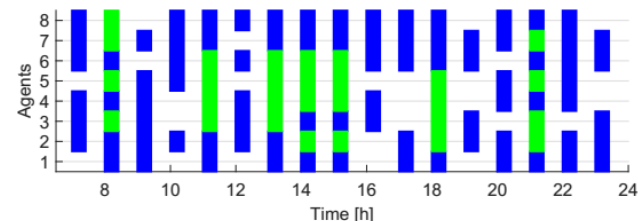
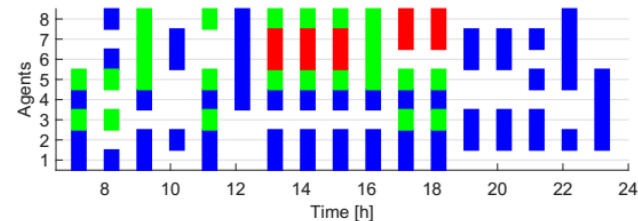
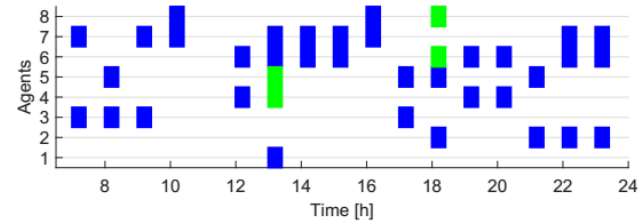
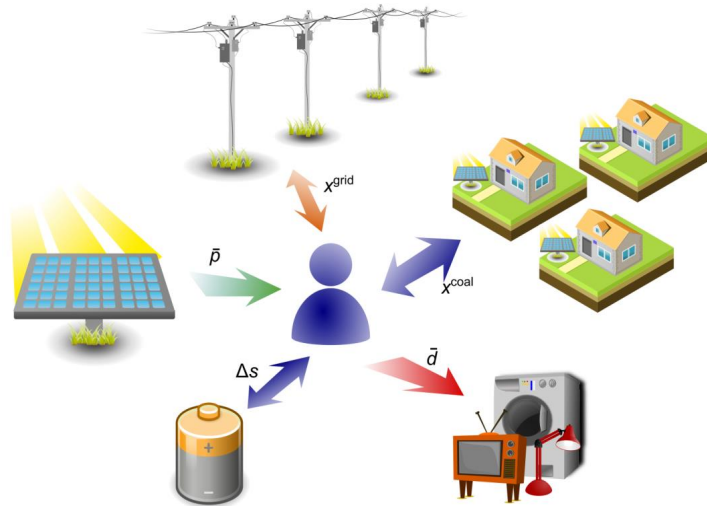
Step increase in the offtakes



Example (IV)

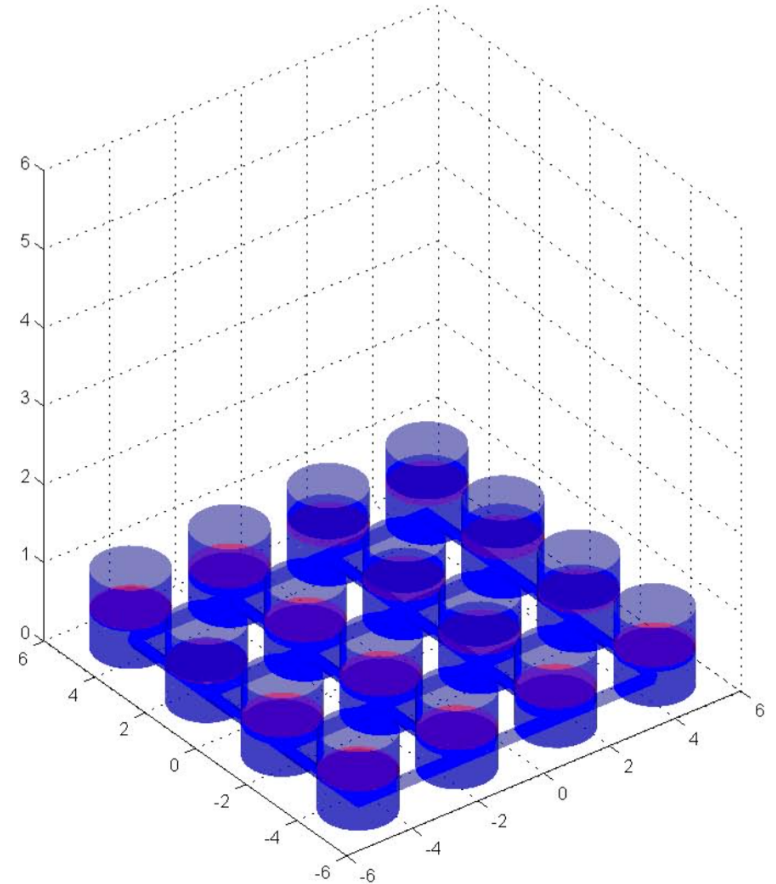


Example (V)



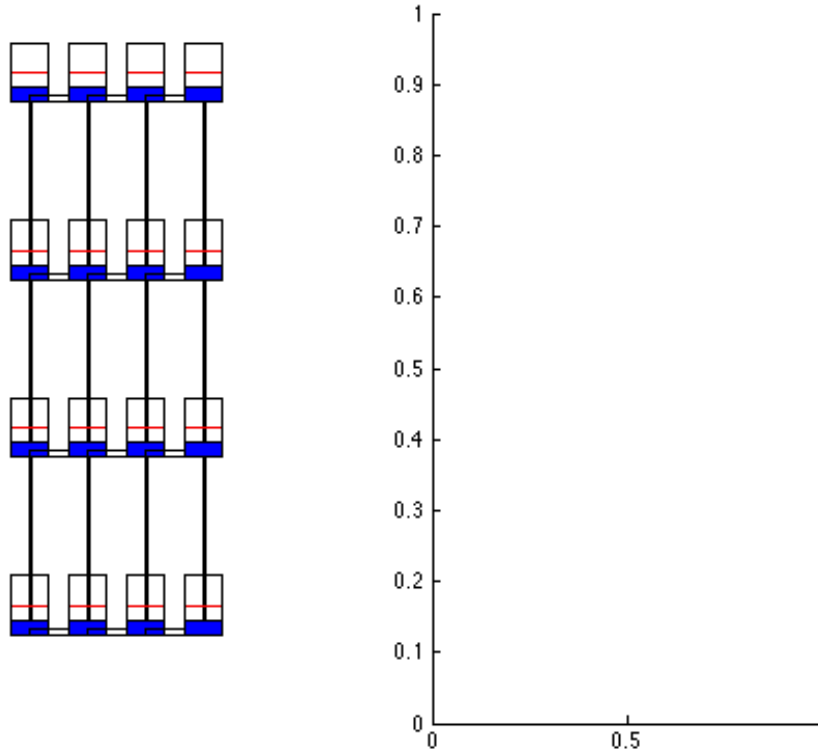
Example (VI)

- 16 interconnected water tanks
 - Only one global source (top-left)
 - Only one global sink (bottom-right)



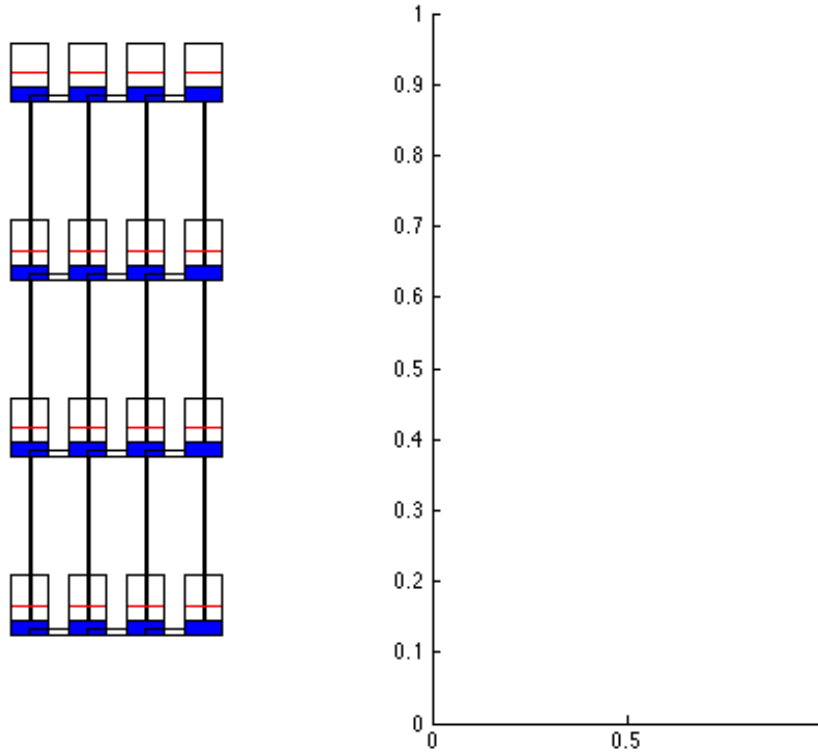
Example (VI)

- Decentralized MPC



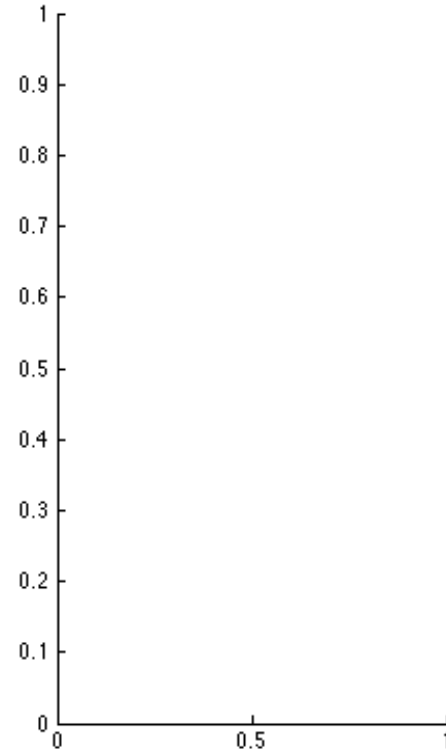
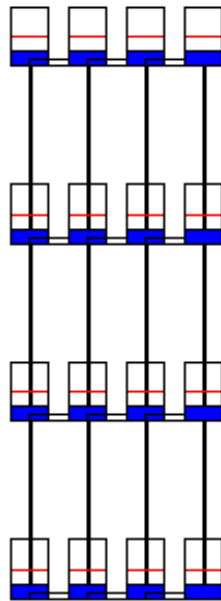
Example (VI)

- Distributed/Centralized MPC

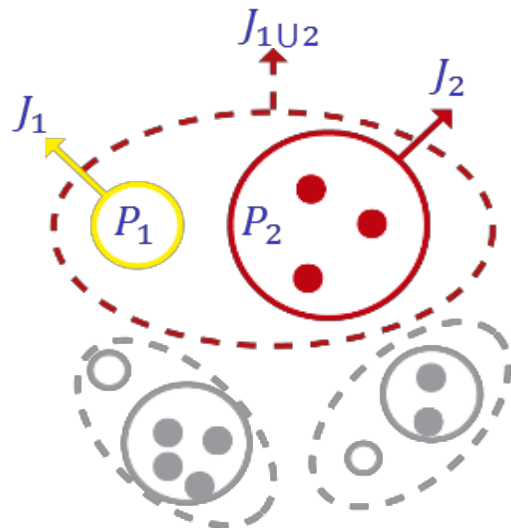


Example (VI)

- Coalitional MPC

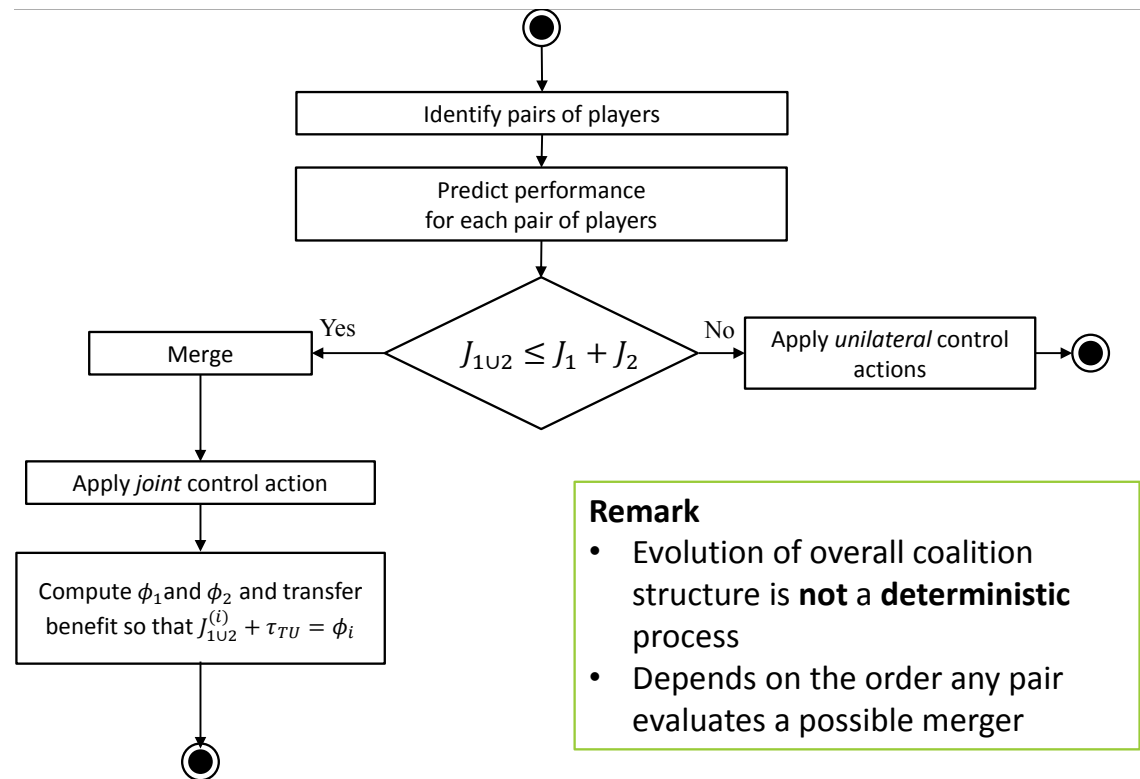


Coalitional Model Predictive Control



Pairwise bargaining with:

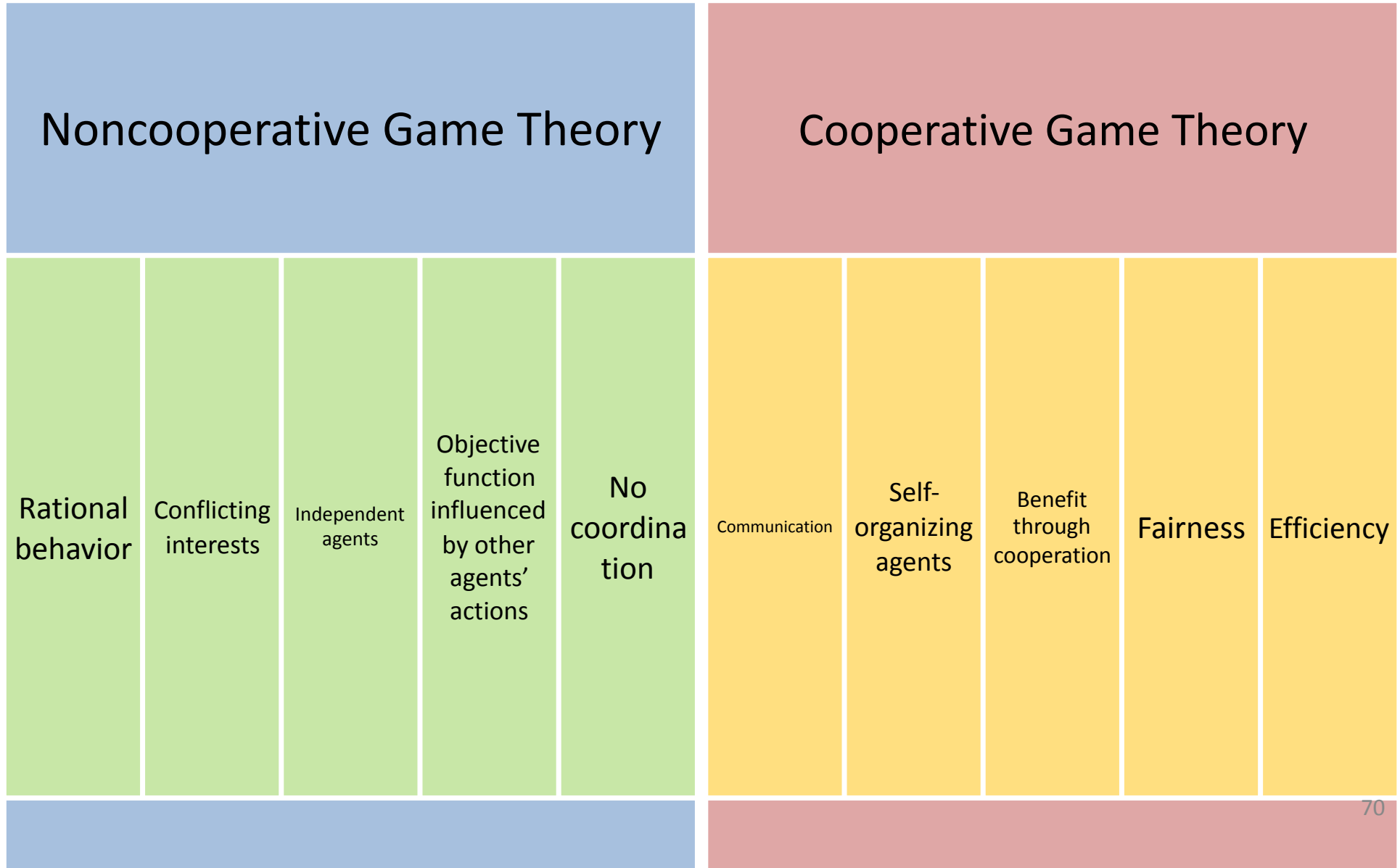
- Closed loop stability
- Coalitional stability
- Convergence to the core



Remark

- Evolution of overall coalition structure is **not a deterministic** process
- Depends on the order any pair evaluates a possible merger

Coalitional Model Predictive Control



Coalitional Model Predictive Control

- Constraints on Shapley value and closed-loop stability

F. J. Muros, J. M. Maestre, E. Algaba, T. Alamo, Eduardo F. Camacho. Networked Control Design for Coalitional Schemes using Game-Theoretic Methods. *Automatica* 78: 320-332, April 2017.

- Computation of a priori and a posteriori values for agents and links with constraints

F. J. Muros, E. Algaba, J. M. Maestre, E. F. Camacho. Harsanyi Power Solutions in Coalitional Control Systems. *IEEE Transactions on Automatic Control* 62(7): 3369-3381, 2017.

- Amalgamation in games and inclusion of constraints with the Banzhaf value

F. J. Muros, E. Algaba, J. M. Maestre, E. F. Camacho. The Banzhaf Value as a Design Tool in Coalitional Control. *Systems and Control Letters* 104: 21-30, June 2017.

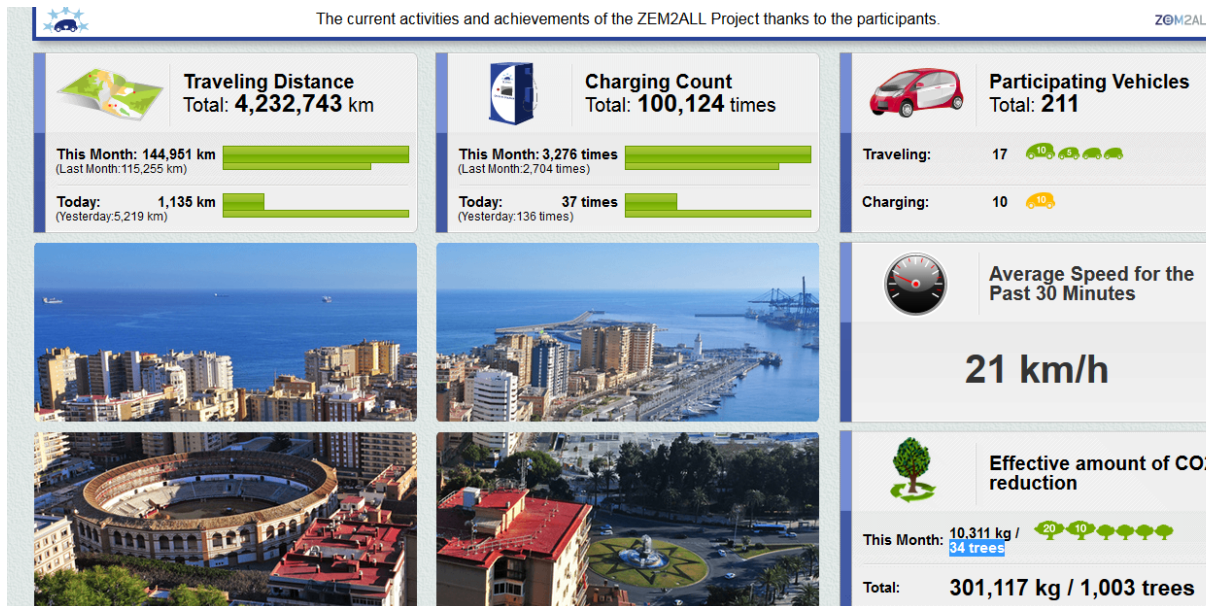
- Application to partitioning and planning problems

F. J. Muros, J. M. Maestre, C. Ocampo, E. Algaba, E. F. Camacho. Partitioning of Large-Scale Systems using Game-Theoretic Coalitional Methods. Accepted in ECC 2018.

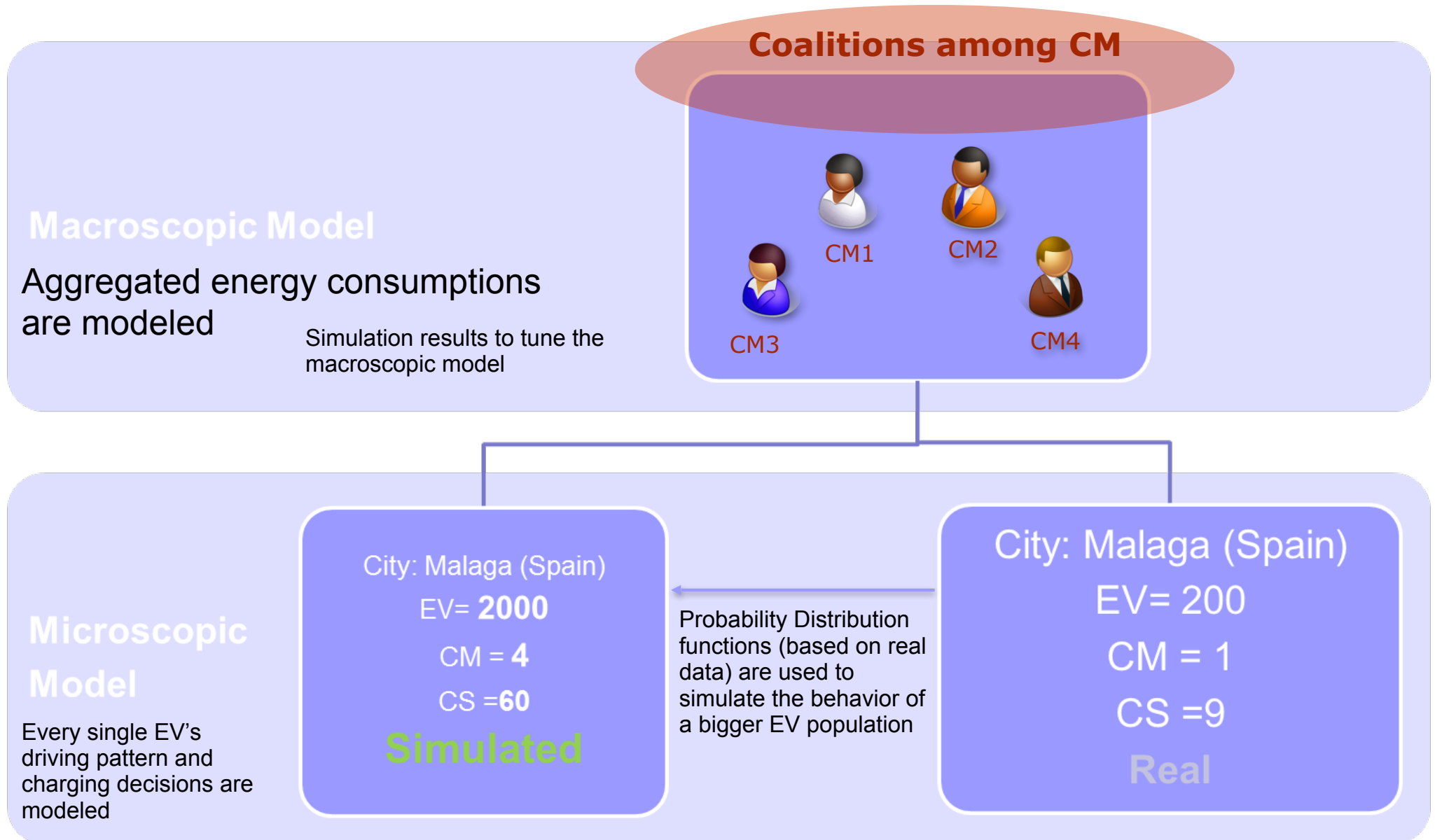
L. A. Fletscher, J. M. Maestre, C. Valencia. Coalitional Planning for Energy Efficiency of HetNets Powered by Hybrid Energy Sources. *IEEE Transactions on Vehicular Technology*. In press.

Example (VII)

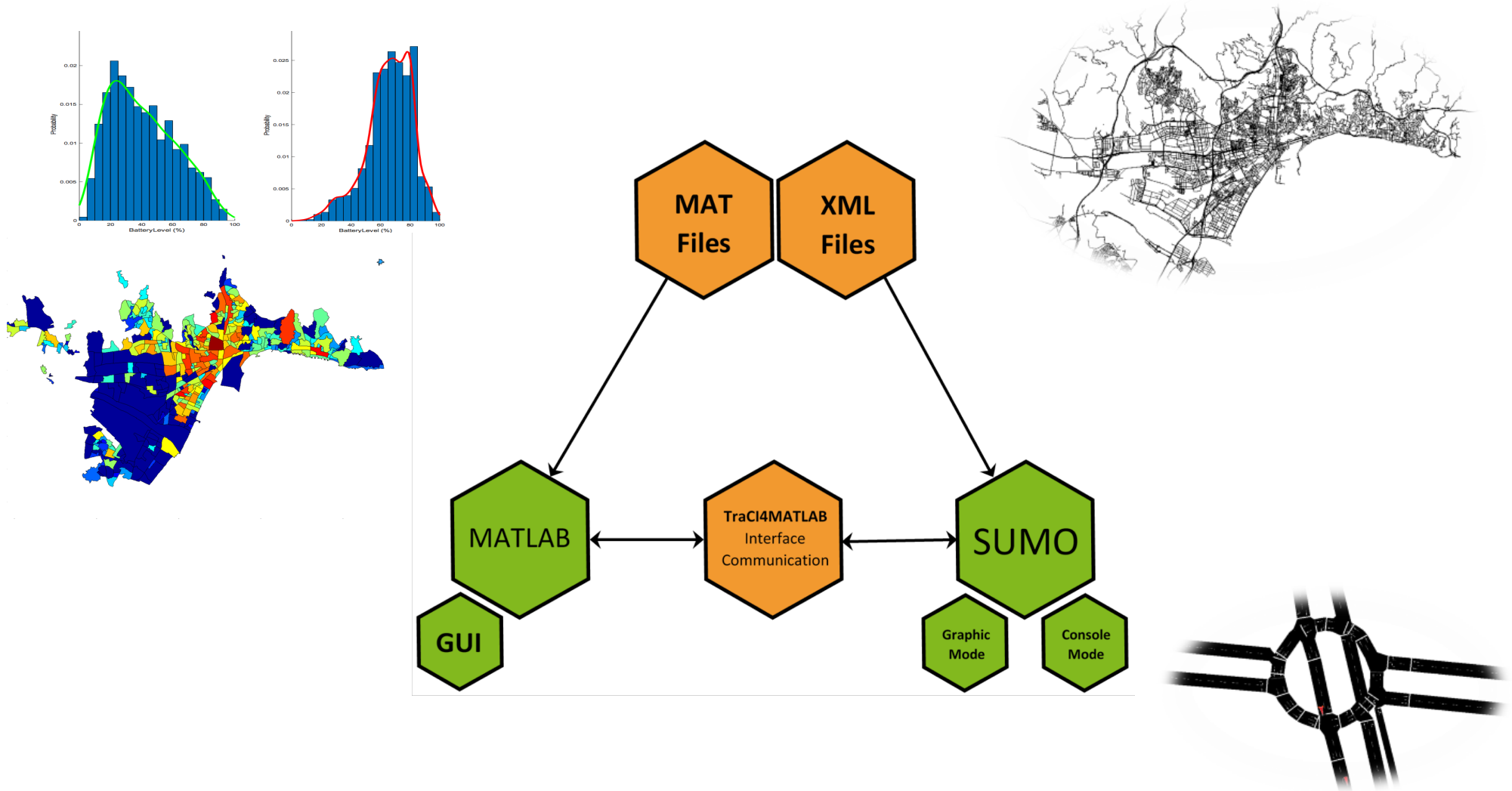
- AYESA / University of Seville

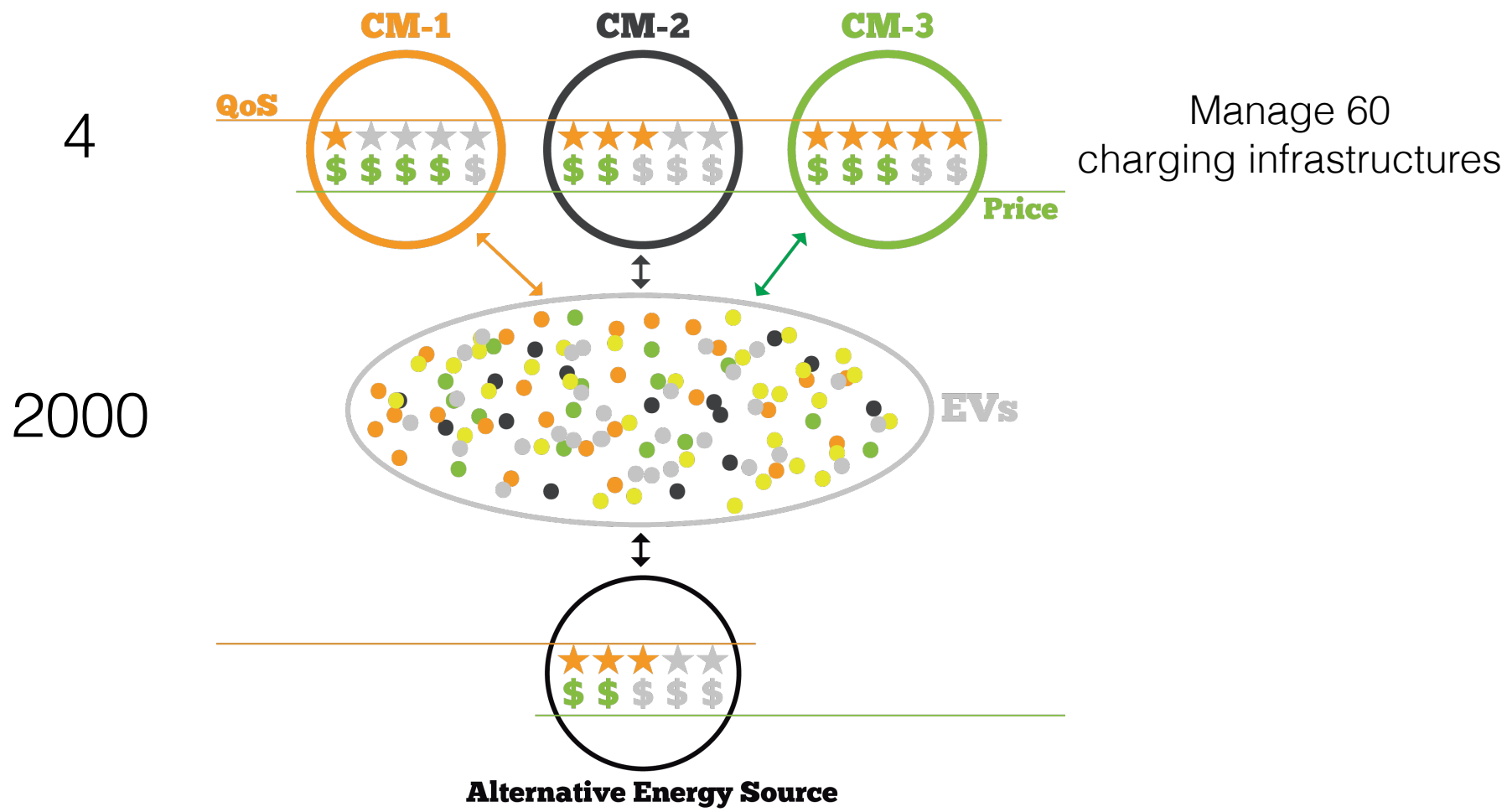


Example (VII)



Example (VII)





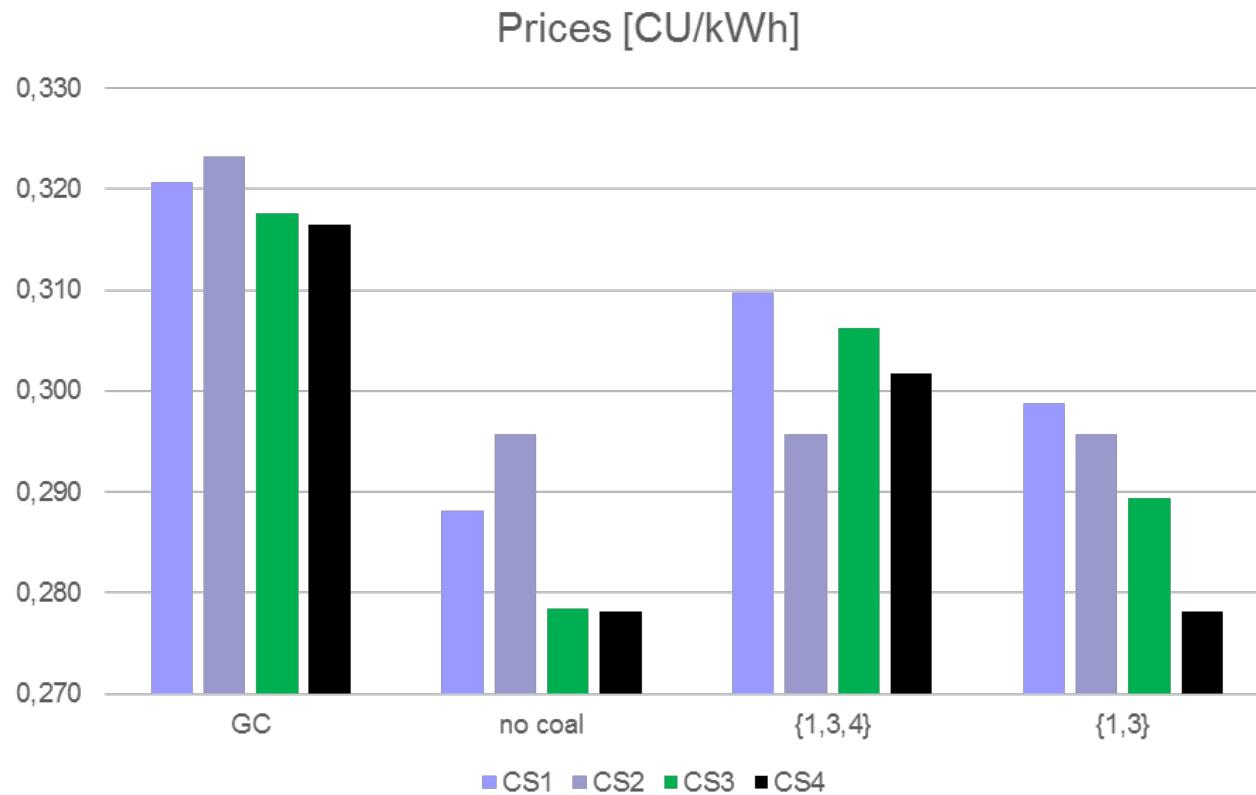
Example (VII)

Basic equation: energy demand at CM-i

- $$E_{t+1}^i = E_t^i + \Delta T E_i \left(\mathbf{b}(p^{per} - p_i) + \mathbf{d}(p^{alt} + \phi - p_i) + \mathbf{c}(QoS_t^i - QoS_t^{per}) - \mathbf{f}(1 - QoS_t^i) \right)$$

Where:

- E_i Energy served by CM-i.
- p_i Energy price offered by CM-i (€/kWh).
- $p^{per} \rightarrow$ Price perceived by customers
- QoS_{per} Quality of Service perceived by customers which is assumed to be a weighted sum of the quality of the services provided by the different CMs.
- p^{alt} is the price of an alternative energy source (e.g. the price for charging the battery at home).
- Δt sample time.
- b_i, c_i, d_i, f_i coefficients: **to be tuned using the microscopic model**



- Coalitions on pricing strategies
 - Measures like limitations on the allowed market share of a single coalition (antitrust) demonstrate to be a mean to keep prices under control
 - Access costs as a function of the sold energy volume allow to avoid the formation of monopoly

Outline

- Model Predictive Control
- Distributed Model Predictive Control
- Coalitional Model Predictive Control
- **Conclusions**

Conclusions

- Model Predictive Control is a powerful and versatile framework
 - i. Mature control method with many theoretical results
 - ii. It largely benefits from the advances on ICT
 - iii. Promising application to cyber-physical systems
- Distributed MPC is powerful but...
 - i. Schemes have been developed naively
 - ii. There is a need for cyber-security
 - iii. Deep implications regarding theoretical properties
- Future points to flexible and resilient architectures
 - i. Misbehaving agents have to be identified
 - ii. Control architecture must be adapted in real time
 - iii. Coalitional control offers an opportunity
- Current work: flexible/coalitional MPC schemes that deal with noncompliant agents

Acknowledgements

- F. Fele, E. F. Camacho, F. Muros, and P. Velarde, who provided part of the slides for this presentation
- 7th FP Project DYMASOS (Dynamic Management of Physically Coupled Systems of Systems)

Thank you!

Questions, suggestions, comments?

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